11. a) Explain the Bearing modulus.
    b) Derive Petroff's equation for a lightly loaded bearing.
    c) A 75mm long full journal bearing of diameter 75mm supports a radial load of 12kN at the shaft speed of 1800 rpm. Assume ratio of diameter to the diametral clearance as 1000. the viscosity of oil is 0.01 Pas at the operating temperature. Determine
       i) Sommerfeld number.
       ii) Coefficient of friction based on Mckee's equation.
       iii) Amount of heat generated **VTU, Dec. 06/ Jan. 2007**
12. a) Explain the significance of the bearing characteristic number in the design of sliding contact bearings.
    b) A full journal bearing of 60 mm diameter and 100 mm long has a bearing pressure of 1.4 N/mm^2. The speed of the journal is 800 rpm and the ratio of journal diameter to the diametral clearance is 1000. The bearing is lubricated with oil whose absolute viscosity at the operating temperature of 75°C may be taken as 0.011 kg/ms. The room temperature is 30°C. Find: i) The amount of artificial cooling required, and ii) The mass of the lubricating oil required, if the difference between the outlet and inlet temperature of the oil is 10°C. Take specific heat of the oil as 1850 J/kg/°C. **VTU, July 2007**
13. a) Explain with sketch theory of hydrodynamic lubrication
    b) Design a full journal bearing subjected to 6000 N at 1000 rpm of the journal. The journal is of hardened steel and the bearing is of babbit metal. The bearing is operating with SAE 40 oil at 70°C and the ambient temperature is 30°C. Also determine the amount of artificial cooling required. **VTU, Dec. 07/ Jan. 2008**
14. a) What is bearing modulus? Explain the significance of bearing modulus in the design of bearing.
    b) Design a journal bearing for a centrifugal pump from the following data:
       Load on the journal = 10kN
       Speed of the journal = 900 rpm
       Ambient temperature = 15°C. **VTU, June/July 2008**
15. a) Explain with a neat sketch, the importance of bearing characteristic number in design of journal bearing.
    b) Design a journal bearing for a centrifugal pump running at 1200 rpm. Diameter of journal is 100 mm and load on bearing is 15 kN. Take l/d = 1.5, bearing temperature 50° and ambient temperature 30°. Find whether artificial cooling is required.

VTU, Dec. 08/ Jan. 2009

UNIT 8

Design of Belts, Ropes and Chains

8.1 INTRODUCTION

Power is transmitted from the prime mover to a machine by means of intermediate mechanism called drives. This intermediate mechanism known as drives may be belt or chain or gears. Belt is used to transmit motion from one shaft to another shaft with the help of pulleys preferably if the centre distance is long. It is not a positive drive since there is slip in belt drive. Three types of belt drives are commonly used. They are (i) Flat belt drive (ii) V-belt drive (iii) Rope or circular belt drive.

8.2 FLAT BELT DRIVE

When the distance between two pulleys is around 10 meters and moderate power is required then flat belt drive is preferred. This may be arranged in two ways (i) open belt drive (ii) Cross belt drive. When the direction of rotation of both the pulleys are required in the same direction, then we can use open belt drive; if direction of rotation of pulleys are required in opposite direction then cross belt drive is used. The pulley which drives the belt is known as driver and the pulley which follows driver is known as driven or follower.

8.2.1 Merits and Demerits of Feat belt drive

Merits :

(i) Simplicity, low cost, smoothness of operation, ability to absorb shocks, flexibility and efficiency at high speeds.

(ii) Protect the driven mechanism against breakage in case of sudden overloads owing to belt slipping.

(iii) Simplicity of care, low maintenance and service.

(iv) Possibility to transmit power over a moderately long distance.

Demerits :

(i) It is not a positive drive.

(ii) Comparatively large size.

(iii) Stretching of belt calling for resewing when the centre distance is constant.
(iv) Not suitable for short centre distance.
(v) Belt joints reduces the life of the belt.
(vi) High bearing loads and belt stresses.
(vii) Less efficiency due to slip and creep.

8.2.2 Creep in Belts

Consider an open belt drive rotating in clockwise direction as shown in Fig. 8.1. The portion of the belt leaving the driven and entering the driver is known as tight side and portion of belt leaving the driver and entering the driven is known as slack side. During rotating there is an expansion of belt on tight side and contraction of belt on the slack side. Due to this uneven expansion and contraction of the belt over the pulleys, there will be a relative movement (motion) of the belt over the pulleys, this phenomenon is known as creep in belts.

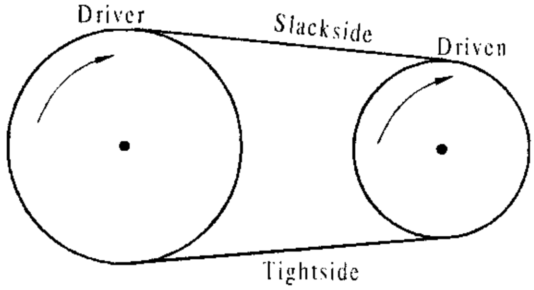


Fig. 8.1

8.2.3 Velocity Ratio

The ratio of angular velocity of the driver pulley to the angular velocity of the driven pulley is known as velocity ratio or speed ratio or transmission ratio

Let d_1 = Speed of driver pulley
d_2 = Speed of driven pulley
n_1 = Speed of driver pulley
n_2 = Speed of driven pulley

Neglecting slip and thickness of belt

Linear speed of belt on driver = Linear speed of belt on driven

i.e., $$\pi d_1 n_1 = \pi d_2 n_2$$

$$\therefore \frac{n_1}{n_2} = \frac{d_2}{d_1}$$

i.e., $$\frac{\text{Speed of driver}}{\text{Speed of driven}} = \frac{\text{Diameter of the driven pulley}}{\text{Diameter of the driver pulley}}$$

Considering the thickness of belt

$$\frac{n_1}{n_2} = \frac{d_2 + t}{d_1 + t}$$

8.2.4 Slip in Belts

Consider an open belt drive rotating in clockwise direction, this rotation of belt over the pulleys is assumed to be due to firm frictional grip between the belt and the pulleys. When this frictional grip becomes in sufficient, there is a possibility of forward motion of driver without carrying belt with it and there is also possibility of belt rotating without carrying the driven pulley with it, this is known as slip in belt. Therefore slip may be defined as the relative motion between the pulley and the belt in it. This reduces velocity ratio and usually expressed as percentage.

8.2.5 Effect of Slip on Velocity Ratio

Let s_1 = Percentage of slip between driver pulley rim and the belt.

s_2 = Percentage of slip between the belt and the driven pulley rim.

Linear speed of driver $= \pi d_1 n_1$

$$\therefore \text{ linear speed of belt} = \pi d_1 n_1 - \frac{\pi d_1 n_1 s_1}{100} = \pi d_1 n_1 \left(1 - \frac{s_1}{100}\right)$$

$$\text{Hence, speed of driven} = \pi d_1 n_1 \left(1 - \frac{s_1}{100}\right)\left(1 - \frac{s_2}{100}\right)$$

$$\text{i.e., } \pi d_2 n_2 = \pi d_1 n_1 \left(1 - \frac{s_1}{100}\right)\left(1 - \frac{s_2}{100}\right)$$

$$\therefore \text{ Velocity ratio } \frac{n_1}{n_2} = \frac{d_2}{d_1\left(1 - \frac{s_1}{100}\right)\left(1 - \frac{s_2}{100}\right)}$$

$$= \frac{d_2}{d_1\left(1 - \frac{s_1 + s_2}{100}\right)} = \frac{d_2}{d_1\left(1 - \frac{s}{100}\right)} \quad \text{Neglect } \frac{s_1 . s_2}{10000} \text{ since very small}$$

Where s = Total percentage slip $= s_1 + s_2$

$$\text{Considering thickness velocity ratio } \frac{n_1}{n_2} = \frac{d_2 + t}{(d_1 + t)\left(1 - \frac{s}{100}\right)}$$

8.2.6 Materials Used for Belt

Belts used for power transmission must be strong, flexible, durable and must have a high coefficient of friction. The most common belt materials are leather, fabric, rubber, balata, Camel's hair and woven cotton.

8.2.7 Length of Open Belt

Consider an open belt drive as shown in Fig. 8.2. Let D = diameter of larger pulley; d = diameter of smaller pulley; C = distance between centres of pulley; L = length of belt.

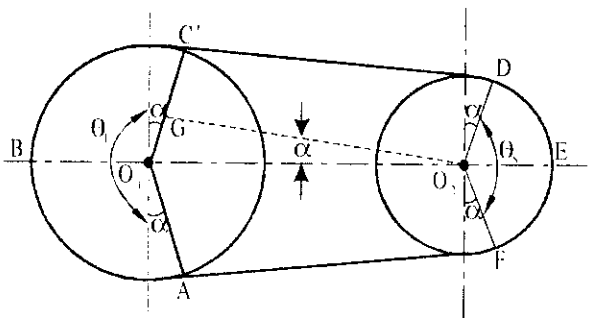


Fig. 8.2

Length of open belt = Arc AB + Arc BC' + C'D + Arc DE + Arc EF + FA

$$\therefore \; L = \sqrt{4C^2 - (D-d)^2} + \frac{1}{2}(D\,\theta_L + d\theta_s) \qquad \text{---- 21.7}$$

Angle of contact on the larger pulley

$$\theta_L = \pi + \left\{2\sin^{-1}\left(\frac{D-d}{2C}\right)\right\}\frac{\pi}{180} \qquad \text{---- 21.10a}$$

Angle of contact on the smaller pulley

$$\theta_s = \pi - \left\{2\sin^{-1}\left(\frac{D-d}{2C}\right)\right\}\frac{\pi}{180} \qquad \text{---- 21.10b}$$

Where θ_L and θ_s are in radians. For equal diameter pulleys $\theta_L = \theta_s = \pi$ radians.

For unequal diameter pulleys, since slip will occur first on the smaller diameter pulley, it is necessary to consider θ_s while designing the belt.

8.2.8 Length of Cross Belt

Consider a cross-belt drive as shown in Fig. 8.3 Let D = diameter of larger pulley; d = diameter of smaller pulley; L = Length of belt.

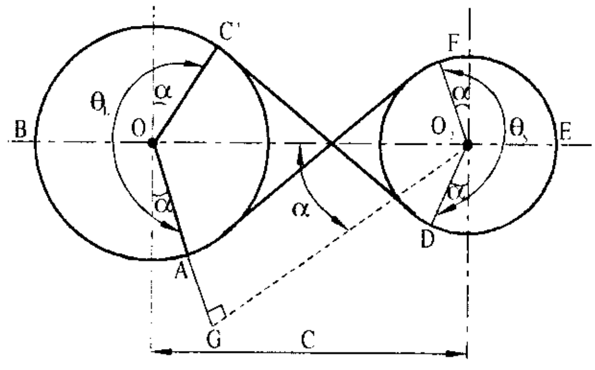


Fig. 8.3

Length of cross belt = Arc AB + Arc BC' + C'D + Arc DE + Arc EF + FA

$$\therefore\ L = \sqrt{4C^2 - (D+d)^2} + \frac{\theta}{2}(D+d) \quad \text{---- 21.8}$$

In cross belt $\theta_L = \theta_s = \theta$

$$\therefore\ \text{Angle of contact } \theta = \pi + \left\{2\sin^{-1}\left(\frac{D+d}{2C}\right)\right\}\frac{\pi}{180} \quad \text{---- 21.10 c}$$

Where θ in radians.

8.2.9 Ratio of Belt Tensions

Consider a driven pulley rotating in clockwise direction as shown in Fig. 8.4

Let T_1 = Tension on tight side

T_2 = Tension on slack side

θ = Angle of lap

R_N = Normal reaction

F = Frictional force = μR_N.

Now consider a small elemental portion of the belt PQ subtending an angle $\delta\theta$ at the centre. The portion of the belt PQ is in equilibrium under the action of the following forces. (i) Tension T at P (ii) Tension T + δT at Q (iii) Normal reaction R_N (iv) Frictional force $F = \mu R_N$

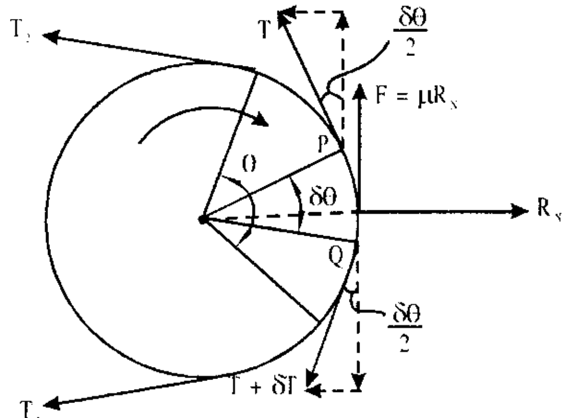


Fig. 8.4

Resolving the forces horizontally

$$R_N = T\sin\frac{\delta\theta}{2} + (T+\delta T)\sin\frac{\delta\theta}{2}$$

$$= T\delta\theta\left[\text{Since small, } \sin\frac{\delta\theta}{2} \approx \frac{\delta\theta}{2} \text{ and neglect } \delta_T.\frac{\delta\theta}{2}\right] \quad \text{---- (i)}$$

Resolving the forces vertically

$$\mu R_N + T\cos\frac{\delta\theta}{2} = (T+\delta_T)\cos\frac{\delta\theta}{2}$$

$$\therefore\ R_N = \frac{\delta_T}{\mu}\left[\text{Since } \frac{\delta\theta}{2} \text{ is small, } \cos\frac{\delta\theta}{2} \approx 1\right] \quad \text{---- (ii)}$$

Equating (i) and (ii)

$$\frac{\delta_T}{\mu} = T\delta\theta$$

$$\text{i.e., } \frac{\delta_T}{T} = \mu\delta\theta$$

Integrating between their respective limits

$$\text{i.e., } \int_{T_2}^{T_1} \frac{\delta_T}{T} = \int_0^{\theta} \mu\delta\theta$$

$$\therefore\ \frac{T_1}{T_2} = e^{\mu\theta} \text{ where } \theta \text{ in radians.}$$

8.2.10 Centrifugal Tension

Consider a driver pulley rotating in clockwise direction, because of rotation of pulley there will be centrifugal force which acts away from the pulley. The tensions created because of this centrifugal force both on tight and slack side is known as centrifugal tension.

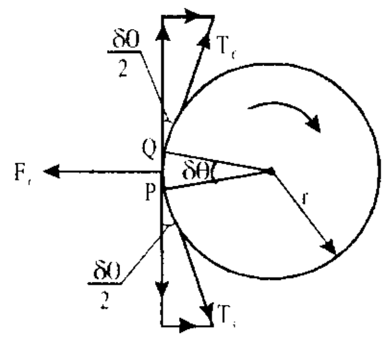


Fig. 8.5

Let m = Mass of belt per meter length

v = Velocity in m/sec

T_c = Centrifugal tension in N

r = Radius of pulley;

F_c = Centrifugal force

Consider a small elemental portion of the belt PQ subtending an angle $\delta\theta$ at the centre as shown in Fig. 8.5.

Now the mass of belt PQ = M = Mass per unit length × Are length PQ = mrdθ

Centrifugal force at the elemental portion PQ = $F_c = \dfrac{Mv^2}{r} = \dfrac{mr\delta\theta . v^2}{r} = mv^2\delta\theta$ ---- (i)

Also resolving the forces horizontally,

$$F_c = T_c \sin\frac{\delta\theta}{2} + T_c \sin\frac{\delta\theta}{2}$$

$$= 2\,T_c\,\frac{\delta\theta}{2}\left[\text{Since } \frac{\delta\theta}{2} \text{ is small, } \sin\frac{\delta\theta}{2} \approx \frac{\delta\theta}{2}\right] \qquad \text{---- (ii)}$$

Equating equations (i) and (ii)

$$2T_c\frac{\delta\theta}{2} = mv^2\delta\theta$$

$$\therefore\ T_c = mv^2$$

Considering centrifugal tension,

Tension on tight side = $T_1 + T_c$; Tension on slack side = $T_2 + T_c$.

8.2.11 Effect of Centrifugal Tension on Ratio of Tensions

Ratio of belt tension considering the effect of centrifugal tension is $\dfrac{T_1 - T_c}{T_2 - T_c} = e^{\mu\theta}$

8.2.12 Power transmitted by belt drive

$$\text{Power } N = \frac{(T_1 - T_2).v}{1000}\ \text{kW}$$

where $T_1 - T_2$ = Effective tension in Newtons and v = Velocity in m/sec.

8.2.13 Effect of Centrifugal Tension on Power

$$\text{We have } N = \frac{(T_1 - T_2)v}{1000}\ \text{kW}$$

Ratio of tension considering centrifugal tension $\dfrac{T_1 - T_c}{T_2 - T_c} = e^{\mu\theta}$

$$\therefore\ T_2 = \frac{T_1 - T_c}{e^{\mu\theta}} + T_c = \frac{T_1}{e^{\mu\theta}} + T_c\left(1 - \frac{1}{e^{\mu\theta}}\right)$$

$$= \frac{T_1}{e^{\mu\theta}} + T_c.k \text{ where } k = 1 - \frac{1}{e^{\mu\theta}}$$

$$\therefore\ N = \frac{\left\{T_1 - \left(\frac{T_1}{e^{\mu\theta}} + T_c.k\right)\right\}v}{1000} = \frac{\left\{T_1\left(1 - \frac{1}{e^{\mu\theta}}\right) - T_c k\right\}}{1000}.v$$

$$= \frac{\{T_1 k - T_c k\} v}{1000} = \frac{\{T_1 - T_c\} kv}{1000} \text{ kW} \qquad \text{---- 21.4a}$$

Neglecting the effect of centrifugal tension

$$N = \frac{T_1 kv}{1000} \text{ kW} \qquad \text{---- 21.2d}$$

8.2.14 Initial Tension

The motion of the belt with the pulleys is assumed to be due to firm frictional grip between the belt and pulley surface. To increase this grip the belt is mounted on the pulleys with some tension when the pulleys are stationary. The tension provided in the belt while mounting on the pulleys is "Initial tension" and is represented by T_0. Since in actual practice the belt is not perfectly elastic, C.G. Barth has given the relation as

$$2\sqrt{T_o} = \sqrt{T_1} + \sqrt{T_2} \qquad \text{---- 21.12}$$

8.2.15 Design Procedure for Flat Belt Drive

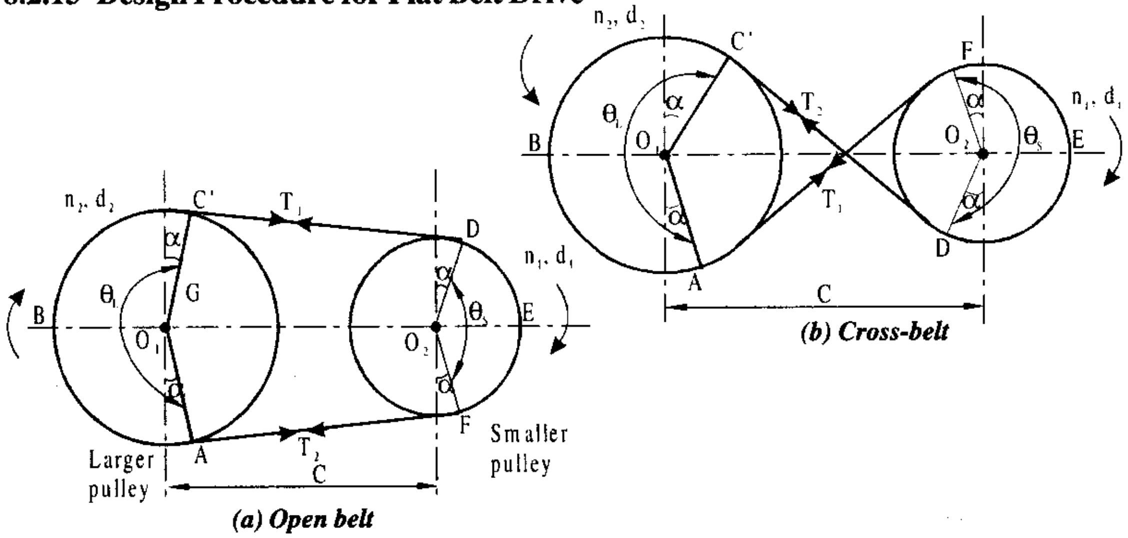


Fig. 8.6

d_1 = Diameter of smaller pulley = d

n_1 = Speed of smaller pulley

d_2 = Diameter of larger pulley = D

n_2 = Speed of larger pulley

C = Centre distance

b = Width of belt

t = Thickness of belt

L = Length of belt

g = Acceleration due to gravity = 9810 mm/sec^2
μ = Coefficient of friction
w = Specific weight of belt material = 10×10^{-6} N/mm^2 for leather
σ_1 = Allowable or safe stress on the tight side
σ_2 = Slack side stress
σ_c = Centrifugal stress
v = Velocity of belt = $\dfrac{\pi dn}{60{,}000}$ m/sec, where d in mm
θ_S = Angle of contact for smaller pulley } For open belt
θ_L = Angle of contact for larger pulley }
θ = Angle of contact for cross-belt
T_1 = Tension on tight side
T_2 = Tension on slack side
T_c = Centrifugal tension
T_0 = Initial tension

1. Unknown diameter or speed

$$n_1 d_1 = n_2 d_2$$

i.e., $n_1 d = n_2 D$ using the above relation find the unknown value.

2. Velocity

$$v = \frac{\pi d_1 n_1}{60{,}000} \text{ or } \frac{\pi (d_1 + t) n_1}{60{,}000} \text{ or } \frac{\pi (d_2 + t) n_2}{60{,}000}$$

3. Centrifugal stress

$$\sigma_c = \frac{w}{g} . v^2 \times 10^6 \text{ N/mm}^2 \qquad \text{---- 21.3b (DDHB)}$$

where $w = 10 \times 10^{-6}$ N/mm^3 for leather or σ_c from Table 21.3

4. Capacity

Calculate $e^{\mu_L \theta_L}$ and $e^{\mu_s \theta_s}$ and take the smaller value as the capacity. If the coefficient of friction is same for both pulleys then find only $e^{\mu \theta_s}$ since it is smaller than $e^{\mu \theta_L}$.

$$\theta_L = \pi + \left\{ 2\sin^{-1}\left(\frac{D-d}{2C}\right) \right\} \frac{\pi}{180} \qquad \text{---- 21.10a (DDHB)}$$

$$\theta_s = \pi - \left\{ 2\sin^{-1}\left(\frac{D-d}{2C}\right) \right\} \frac{\pi}{180} \qquad \text{---- 21.10b (DDHB)}$$

For cross-belt

$$\theta = \pi + \left\{ 2\sin^{-1}\left(\frac{D+d}{2C}\right) \right\} \frac{\pi}{180} \qquad \text{---- 21.10c (DDHB)}$$

5. *Find 'k'*

$$k = \frac{e^{\mu\theta} - 1}{e^{\mu\theta}}$$

6. *Width of belt*

Power transmitted per mm^2 area $= \frac{(\sigma_l - \sigma_c)kv}{1000} \frac{kW}{mm^2}$

where σ_l in N/mm^2; σ_c in N/mm^2 and v in m/sec

Area of cross section of belt $A = \frac{\text{Total given power}}{\text{Power transmitted / mm}^2}$

Also $A = b \times t \therefore b =$ width of belt

7 . *Length of belt*

For open belt

$$L = \sqrt{4C^2 - (D-d)^2} + \frac{1}{2}(D\theta_L + d\theta_s) \qquad \text{---- 21.7 (DDHB)}$$

For cross belt

$$L = \sqrt{4C^2 - (D+d)^2} + \frac{\theta}{2}(D + d) \qquad \text{----21.8 (DDHB)}$$

8. *Initial tension in the belt*

$$2\sqrt{T_o} = \sqrt{T_1} + \sqrt{T_2} \qquad \text{---- 21.12 (DDHB)}$$

$$T_1 = \sigma_1 A$$

$$T_2 = \sigma_2 A$$

σ_2 is obtained using the following relation.

$$\frac{\sigma_1 - \sigma_C}{\sigma_2 - \sigma_C} = e^{\mu\theta} \qquad \text{---- 21.3a (DDHB)}$$

Example 8.1

A belt is required to transmit 18.5 kW from a pulley of 1.2 m diameter running at 250 rpm to another pulley which run at 500 rpm. The distance between the centres of pulleys is 2.7 m. The following data refer to an open belt drive $\mu = 0.25$. Safe working stress for leather is 1.75 N/mm². Thickness of belt = 10 mm. Determine the width and length of belt taking centrifugal tension into account. Also find the initial tension in the belt and absolute power that can be transmitted by this belt and the speed at which this can be transmitted.

Data :

Open belt drive; N = 18.5 kW; n_1 = 500 rpm = Speed of smaller pulley;

d_2 = 1.2 m = 1200 mm = D = Diameter of larger pulley; n_2 = 250 rpm = Speed of larger pulley;

C = 2.7 m = 2700 mm; μ = 0.25; σ_1 = 1.75 N/mm²; t = 10 mm

Solution :

(i) Diameter of smaller pulley

$$n_1 d_1 = n_2 d_2$$

$$500 \times d_1 = 250 \times 1200$$

$$\therefore \text{Diameter of smaller pulley } \mathbf{d_1} = \mathbf{600\ mm = d}$$

(ii) Velocity

$$v = \frac{\pi(D+t)n_2}{60{,}000} = \frac{\pi(1200+10)250}{60{,}000} = 15.839 \text{ m/sec.} \qquad (\because \text{ Larger pulley is the driver})$$

(iii) Centrifugal stress

$$\sigma_C = \frac{wv^2}{g} \times 10^6 \qquad \text{---- 21.3b (DDHB)}$$

Assume specific weight of leather as 10×10^{-6} N/mm³

$$\therefore\ \sigma_C = \frac{10 \times 10^{-6}}{9810} \times 15.839^2 \times 10^6 = 0.25573 \text{ N/mm}^2$$

(iv) Capacity

Since coefficient of friction is same for both smaller and larger pulleys, capacity = $e^{\mu\theta_s}$

$$\text{i.e., } e^{\mu\theta} = e^{\mu\theta_s}$$

$$\theta_s = \pi - \left\{2\sin^{-1}\left(\frac{D-d}{2C}\right)\right\}\frac{\pi}{180} \qquad \text{---- 21.10b (DDHB)}$$

$$= \pi - \left\{2\sin^{-1}\left(\frac{1200-600}{2\times 2700}\right)\right\}\frac{\pi}{180} = 2.92 \text{ radians}$$

$$\therefore\ e^{\mu\theta} = e^{0.25\times 2.92} = 2.075$$

(v) Constant

$$k = \frac{e^{\mu\theta}-1}{e^{\mu\theta}} = \frac{2.075-1}{2.075} = 0.52$$

(vi) Width of belt

$$\text{Power transmitted per mm}^2\text{ area} = \frac{(\sigma_1 - \sigma_c)kv}{1000} \qquad \text{---- 21.4a (DDHB)}$$

$$= \frac{(1.75-0.25573)0.52\times 15.839}{1000} = 0.01231 \text{ kW}$$

$$\therefore\ \text{Area of cross section of belt} = \frac{\text{Total given power}}{\text{Power transmitted per mm}^2\text{ area}} = \frac{18.5}{0.01231} = 1503.18 \text{ mm}^2$$

$$\text{Also } A = b \times t$$

$$1503.18 = b \times 10$$

$$\therefore\ \mathbf{b} = \mathbf{150.318\ mm} \qquad \text{---- From Table 21.7 (DDHB)}$$

$$\text{Standard width } b = 152 \text{ mm}$$

(vii) Length of belt

$$\text{Length of open belt } L = \sqrt{4C^2-(D-d)^2} + \frac{1}{2}(D\theta_L + \theta_s d) \qquad \text{----21.7 (DDHB)}$$

$$\theta_L = \pi + \left\{2\sin^{-1}\left(\frac{D-d}{2C}\right)\right\}\frac{\pi}{180} \qquad \text{----21.10a (DDHB)}$$

$$= \pi + \left\{2\sin^{-1}\left(\frac{1200-600}{2\times 2700}\right)\right\}\frac{\pi}{180} = 3.364 \text{ radians}$$

$$L = \sqrt{4\times 2700^2-(1200-600)^2} + \frac{1}{2}(3.364\times 1200 + 2.92\times 600)$$

$$\therefore \mathbf{L = 8260.96\ mm}$$

(viii) Initial tension

$$2\sqrt{T_o} = \sqrt{T_1} + \sqrt{T_2} \qquad \text{----21.12 (DDHB)}$$

$$T_1 = \sigma_1 A = 1.75\times 1503.18 = 2630.566\text{ N}$$

$$\frac{\sigma_1-\sigma_C}{\sigma_2-\sigma_C} = e^{\mu\theta};\ \frac{1.75-0.25573}{\sigma_2-0.25573} = 2.075;\ \therefore\ \sigma_2 = 0.97586\text{ N/mm}^2$$

$$T_2 = \sigma_2 A = 0.97586\times 1503.18 = 1466.894\text{ N}$$

$$2\sqrt{T_o} = \sqrt{2630.566} + \sqrt{1466.894}$$

$$\therefore \mathbf{T_o = 2006.552\ N}$$

(ix) Absolute power

For maximum power transmission

$$\sigma_C = \frac{\sigma_1}{3} = \frac{1.75}{3} = 0.5833\text{ N/mm}^2$$

$$\text{Also } \sigma_C = \frac{w}{g}v^2\times 10^6$$

$$\therefore 0.5833 = \frac{10\times 10^{-6}}{9810}\times v^2\times 10^6$$

$$\therefore \mathbf{v = 23.92\ m/sec}$$

$$\therefore \text{Power transmitted} \setminus \text{mm}^2 = \frac{(\sigma_1-\sigma_C)kv}{1000}$$

$$= \frac{(1.75-0.5833)0.52\times 23.92}{1000}$$

$$= 0.0145\text{ kW}$$

$$\therefore \text{Total absolute power} = \text{Area of c/s of belt}\times\text{power per mm}^2$$

$$= 1503.18\times 0.0145 = 21.7961\text{ kW}$$

$$\therefore \text{Absolute power} = 21.8\text{ kW.}$$

Example 8.2

A flat belt is required to transmit 10 kW from a pulley of 600 mm effective diameter running at 300 rpm. The angle of contact is spread over 7/16 of circumference. Determine the width of belt whose thickness is 10 mm. The allowable stress for the belt is 2.25 N/mm². Coefficient of friction between the pulley and belt is 0.3.

Data :

$$N = 10\,kW;\ d = 600\,mm;\ n = 300\,rpm;$$

$$\theta = \frac{7}{16} \times \text{circumference} = \frac{7}{16} \times 2\pi = 2.75 \text{ radian};\ \mu = 0.3;\ t = 10\,mm;\ \sigma_t = 2.25\,N/mm^2$$

Solution :

(i) Velocity

$$v = \frac{\pi dn}{60{,}000} \ (\because \text{ Effective diameter})$$

$$= \frac{\pi \times 600 \times 300}{60{,}000} = 9.42 \text{ m/sec}$$

(ii) Centrifugal stress

Assume specific weight of leather as 10×10^{-6} N/mm²

$$\therefore \sigma_C = \frac{w}{g} v^2 \times 10^6 \qquad \text{----21.3b (DDHB)}$$

$$= \frac{10 \times 10^{-6} \times 9.42^2 \times 10^6}{9810} = 0.09\,N/mm^2$$

(iii) Capacity

$$\theta_s = 2.75 \text{ radians} = \theta$$

$$\therefore e^{\mu\theta} = e^{0.3 \times 2.75} = 2.28$$

(iv) Constant

$$k = \frac{e^{\mu\theta} - 1}{e^{\mu\theta}} = \frac{2.28 - 1}{2.28} = 0.56$$

(v) Width of belt

$$\text{Power transmitted per mm}^2 \text{ area} = \frac{(\sigma_1 - \sigma_C)kv}{1000} \qquad \text{---- 21.4a (DDHB)}$$

$$= \frac{(2.25 - 0.09)0.56 \times 9.42}{1000} = 0.0114\,kW$$

$$\therefore \text{ Area of cross section of belt} = \frac{\text{Total power}}{\text{Power transmitted per mm}^2 \text{ area}} = \frac{10}{0.0114} = 877.2\,mm^2$$

$$\text{Also } A = b \times t$$

$$877.2 = b \times 10$$

$$\therefore \text{Width of belt } \mathbf{b} = \mathbf{87.72\ mm} \approx 88\text{ mm}$$

Example 8.3

Design a belt drive to transmit 25 kW from a motor shaft rotating at 1500 rpm to a compressor running at 500 rpm. The motor pulley is 96 mm effective diameter and the centre distance between the shafts is 1.5 m.

Data :

$$\mathbf{N = 25\ kW;\ n_1 = 1500\ rpm;\ d_1 = 96\ mm = d;\ n_2 = 500\ rpm}$$

$$\mathbf{C = 1.5\ m = 1500\ mm}$$

Solution :

(i) Diameter of larger pulley

$$n_1 d_1 = n_2 d_2$$

$$1500 \times 96 = 500 \times d_2$$

$$\therefore \text{Diameter of larger pulley } \mathbf{d_2} = \mathbf{288\ mm = D}$$

(ii) Velocity

$$v = \frac{\pi d_1 n_1}{60{,}000} \quad (\because \text{ Effective diameter})$$

$$= \frac{\pi \times 96 \times 1500}{60{,}000} = 7.54 \text{ m/sec}$$

(iii) Centrifugal stress

$$\sigma_c = \frac{w}{g} v^2 \times 10^6 \qquad \text{---- 21.3b (DDHB)}$$

$$\text{Specific weight of leather} = 10 \times 10^{-6}\ \text{N/mm}^3$$

$$\therefore\ \sigma_c = \frac{10 \times 10^{-6}}{9810} \times 7.54^2 \times 10^6 = 0.0579\ \text{N/mm}^2$$

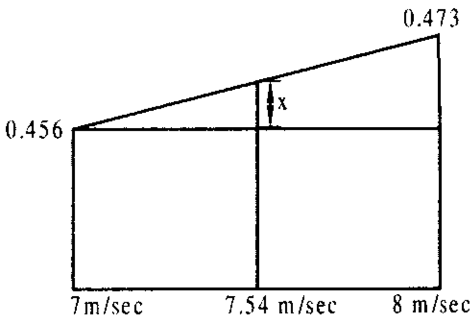
(iv) Capacity

From Table 21.4 (Old DDHB) or Table 21.4A (New DDHB)

$$\text{When } v = 7\ \text{m/sec};\ \mu = 0.456$$

$$v = 8\ \text{m/sec};\ \mu = 0.473$$

By Interpolation



$$\frac{x}{0.473 - 0.456} = \frac{7.54 - 7}{8 - 7}$$

$$\therefore\ x = 9.18 \times 10^{-3}$$

$$\therefore\ \text{when } v = 7.54\ \text{m/sec,}$$

$$\mu = 0.456 + 9.18 \times 10^{-3} = 0.46518$$

$$\text{Capacity} = e^{\mu\theta} = e^{\mu\theta_s}$$

$$\theta_s = \pi - \left\{2\sin^{-1}\left(\frac{D-d}{2C}\right)\right\}\frac{\pi}{180} \quad \text{----21.10b (DDHB)}$$

$$= \pi - \left\{2\sin^{-1}\left(\frac{288-96}{2\times1500}\right)\right\}\frac{\pi}{180} = 3.0135 \text{ radians} = \theta$$

$$\therefore e^{\mu\theta} = e^{0.46518\times3.0135} = 4.0626$$

(v) Constant

$$k = \frac{e^{\mu\theta}-1}{e^{\mu\theta}} = \frac{4.0626-1}{4.0626} = 0.754$$

(vi) Width of belt

From table 21.10 for leather, σ_u = **20.6 N/mm²**

Assume FOS n = 10

$$\therefore \text{Allowable stress} \quad \sigma_1 = \frac{\sigma_u}{n} = \frac{20.6}{10} = 2.06 \text{ N/mm}^2$$

$$\text{Power transmitted per mm}^2 = \frac{(\sigma_1 - \sigma_c)k.v}{1000} \quad \text{---- 21.4a (DDHB)}$$

$$= \frac{(2.06-0.0579)0.754\times7.54}{1000} = 0.01138 \text{ kW}$$

$$\therefore \text{Area of cross section of belt} = \frac{\text{Total power}}{\text{Power transmitted per mm}^2} = \frac{25}{0.01138} = 2196.4 \text{ mm}^2$$

Also A = b × t Assume t = 10 mm; 2196.4 = b × 10

∴ b = **219.64 ≈ 220 mm** = width of belt.

(vii) Length of belt

$$\text{Length of open belt } L = \sqrt{4C^2-(D-d)^2} + \frac{1}{2}(D\theta_L + d\theta_S) \quad \text{----21.7 (DDHB)}$$

$$\theta_L = \pi + \left\{2\sin^{-1}\left(\frac{D-d}{2C}\right)\right\}\frac{\pi}{180} \quad \text{----21.10a (DDHB)}$$

$$= \pi + \left\{2\sin^{-1}\left(\frac{288-96}{2\times1500}\right)\right\}\frac{\pi}{180} = 3.27 \text{ radians}$$

$$\therefore L = \sqrt{4\times1500^2-(288-96)^2} + \frac{1}{2}(288\times3.27 + 96\times3.0135)$$

$$= \mathbf{3609.4\ mm}$$

(viii) Initial tension in the belt

$$2\sqrt{T_o} = \sqrt{T_1} + \sqrt{T_2} \quad \text{----21.12 (DDHB)}$$

$$\frac{\sigma_1 - \sigma_C}{\sigma_2 - \sigma_C} = e^{\mu\theta} \quad \text{----21.3a(DDHB)}$$

$$\therefore \frac{2.06 - 0.0579}{\sigma_2 - 0.0579} = 4.0626$$

$$\therefore \boldsymbol{\sigma_2} = \mathbf{0.5507\ N/mm^2}$$

$$T_1 = \sigma_1 A = 2.06 \times 2196.4 = 4524.6\ N$$

$$T_2 = \sigma_2 A = 0.5507 \times 2196.4 = 1209.6\ N$$

$$2\sqrt{T_0} = \sqrt{4524.6} + \sqrt{1209.6}$$

$$\therefore \mathbf{T_0} = \mathbf{2603.3\ N} = \text{Initial tension}$$

Example 8.4

A belt of 100 mm wide and 10mm thick is transmitting power at 1000 m/min. The net driving tension is 2 times the slack side tension. Allowable stress in the belt material is 2 MPa. Specific weight of the belt material is 10 kN/m³. Determine the power that can be transmitted by the belt. Also determine the absolute power that can be transmitted by the belt and the velocity at which that power can be transmitted.

Data :

$$b = 100\ mm; \quad t = 10\ mm; \quad v = 1000\ m/min = \frac{1000}{60} = 16.67\ m/sec;$$

$$\sigma_1 = 2\ MPa; \quad w = 10\ kN/m^3 = 10 \times 10^{-6}\ N/mm^3;$$

Net tension = 2 × slack side tension.

Solution :

Net tension = Tight side tension - Slack side tension = $(T_1 - T_C) - (T_2 - T_C)$

Since, Net tension = 2 × Slack side tension

$$(T_1 - T_C) - (T_2 - T_C) = 2(T_2 - T_C)$$

$$\text{i.e., } (\sigma_1 - \sigma_C) - (\sigma_2 - \sigma_C) = 2(\sigma_2 - \sigma_C)$$

$$\text{i.e., } \sigma_1 - \sigma_C = 3(\sigma_2 - \sigma_C)$$

$$\therefore \frac{\sigma_1 - \sigma_C}{\sigma_2 - \sigma_C} = 3 = e^{\mu\theta}$$

$$\therefore k = \frac{e^{\mu\theta} - 1}{e^{\mu\theta}} = \frac{3-1}{3} = 0.667$$

$$\text{Centrifugal stress } \sigma_C = \frac{w}{g} v^2 \times 10^6 = \frac{10 \times 10^{-6}}{9810} \times 16.67^2 \times 10^6 = 0.2833\ N/mm^2$$

$$\therefore \text{Power transmitted per mm}^2 \text{ area} = \frac{(\sigma_1 - \sigma_C)k.v}{1000} \quad \text{----21.4a(DDHB)}$$

$$= \frac{(2 - 0.2833)0.667 \times 16.67}{1000} = 0.01908\ kW$$

$$\text{Area of cross section of belt } A = b \times t = 100 \times 10 = 1000\ mm^2$$

$$\therefore \text{ Total power capacity} = \frac{\text{Power}}{\text{mm}^2} \times \text{Area of cross section of belt} = 0.01908 \times 1000$$

$$\therefore \textbf{ Total power capacity N} = \mathbf{19.08\ kW}$$

(ii) Absolute maximum power

For maximum power

$$T_C = \frac{1}{3} T_1$$

$$\therefore \sigma_C = \frac{1}{3}\sigma_1 = \frac{2}{3} = 0.667 \text{ N/mm}^2$$

$$\sigma_C = \frac{wv^2}{g} \times 10^6$$

$$0.667 = \frac{10 \times 10^{-6} \times v^2}{9810} \times 10^6$$

$$\therefore \mathbf{v = 25.573\ m/sec}$$

$$\text{Absolute power per mm}^2 = \frac{(\sigma_1 - \sigma_C)kv}{1000} = \frac{(2-0.667)0.667 \times 25.573}{1000} = 0.02273 \text{ kW}$$

$$\therefore \text{ Total absolute power N} = \frac{\text{Absolute power}}{\text{mm}^2} \times \text{Area of cross section of belt} = 0.02273 \times 1000$$

$$= \mathbf{22.73\ kW}$$

Example 8.5

For a flat belt drive the following data is given,

Speed of motor = 1500 rpm
Speed of driven pulley = 300 rpm
Power to be transmitted = 10 kW
Weight density of leather = 10×10^{-6} N/mm³
Smaller pulley diameter to thickness of belt ratio = 20
Centre distance = 3 m
Allowable stress in the belt = 2.5 MPa
Velocity of belt drive = 15 m/sec
Load factor = 1.2
Coefficient of friction = 0.3

Design a suitable cross-belt drive

Data :

$$n_1 = 1500 \text{ rpm};\ n_2 = 300 \text{ rpm};\ N = 10 \text{ kW};\ w = 10 \times 10^{-6} \text{ N/mm}^3$$

$$\frac{d}{t} = \frac{d_1}{t} = 20;\ C = 3\text{ m} = 3000 \text{ mm};\ \sigma_1 = 2.5 \text{ MPa};\ v = 15 \text{ m/sec};$$

$$\text{Load factor} = 1.2;\ \mu = 0.3;\ \text{Cross-belt drive.}$$

Solution :

(i) Diameter of pulley and thickness of belt

$$v = \frac{\pi(d_1 + t)n_1}{60{,}000} = \frac{\pi\left(d_1 + \frac{d_1}{20}\right)n_1}{60{,}000}$$

$$15 = \frac{\pi \times d_1 \times 1500\left(1 + \frac{1}{20}\right)}{60{,}000}$$

$$d_1 = 181.89\text{mm} \approx 182 \text{ mm} \approx d = \text{diameter of smaller pulley.}$$

$$n_1 d_1 = n_2 d_2$$

$$1500 \times 182 = 300 \times d_2$$

$$\therefore \mathbf{d_2 = 910 \text{ mm} = D} = \text{diameter of larger pulley}$$

$$\text{Since } \frac{d}{t} = 20 \text{ (given)}$$

$$\frac{182}{20} = t = 9.1 \text{ mm}$$

$$\therefore \mathbf{t = 9.1 \text{ mm}} = \text{thickness of belt}$$

(ii) Centrifugal stress

$$\sigma_C = \frac{wv^2}{g} \times 10^6 = \frac{10 \times 10^{-6} \times 15^2}{9810} \times 10^6 = 0.23 \text{ N/mm}^2$$

(iii) Capacity

Since it is cross belt drive, capacity $= e^{\mu\theta}$

$$\theta = \pi + \left\{2\sin^{-1}\left(\frac{D+d}{2C}\right)\right\}\frac{\pi}{180} = 3.5076 \text{ radians}$$

$$\therefore e^{\mu\theta} = e^{0.3 \times 3.5076} = 2.864$$

(iv) Constant

$$k = \frac{e^{\mu\theta} - 1}{e^{\mu\theta}} = \frac{2.864 - 1}{2.864} = 0.6508$$

(v) Width of belt

$$\text{Power transmitted per mm}^2 = \frac{(\sigma_1 - \sigma_C)kv}{1000} \qquad \text{---- 21.4a(DDHB)}$$

$$= \frac{(2.5 - 0.23)0.608 \times 15}{1000} = 0.02216 \text{ kW}$$

$$\therefore \text{Area of cross section of belt} = \frac{\text{Total power}}{\text{Power per mm}^2} \times \text{Load factor} = \frac{10}{0.02216} \times 1.2 = 541.516 \text{ mm}^2$$

$$\text{Also } A = b \times t; \quad \therefore 541.516 = b \times 9.1$$

$$\therefore \mathbf{b} = \mathbf{59.5\ mm} = \text{width of belt.}$$

(vi) Length of belt

$$L = \sqrt{4C^2 - (D+d)^2} + \frac{\theta}{2}(D+d) \quad (\because \text{ Cross belt}) \qquad \text{----21.8 (DDHB)}$$

$$\therefore L = \sqrt{4 \times 3000^2 - (910+182)^2} + \frac{3.5076}{2}(910+182)$$

$$= \mathbf{7815\ mm}$$

(viii) Initial tension

$$2\sqrt{T_o} = \sqrt{T_1} + \sqrt{T_2} \qquad \text{---- 21.12 (DDHB)}$$

$$\frac{\sigma_1 - \sigma_C}{\sigma_2 - \sigma_C} = e^{\mu\theta} \qquad \text{---- 21.3a (DDHB)}$$

$$\text{i.e., } \frac{2.5 - 0.23}{\sigma_2 - 0.23} = 2.864$$

$$\therefore \sigma_2 = \mathbf{1.0226\ N/mm^2}$$

$$\therefore T_1 = \sigma_1 A = 2.5 \times 541.516 = 1353.79\ N$$

$$T_2 = \sigma_2 A = 1.0226 \times 541.516 = 553.753\ N$$

$$\therefore 2\sqrt{T_o} = \sqrt{1353.79} + \sqrt{553.753}$$

$$\therefore \text{Initial tension } \mathbf{T_o} = \mathbf{909.8\ N}$$

Example 8.6

A leather belt 9 mm × 250 mm is used to drive a cast iron pulley 90 cm in diameter at 336 rpm. If the active arc of contact on the smaller pulley is 120° and the stress in tight side 2 MPa, find the power capacity of the belt which weighs 0.00098 kg/cm³. Coefficient of friction of leather on cast iron is 0.35

BU April 99

Data :

$$\mathbf{t = 9\ mm; \quad b = 250\ mm; \quad d = 90\ cm = 900\ mm; \quad n = 336\ rpm;}$$

$$\mathbf{\theta_s = 120°; \quad \sigma_1 = 2\ MPa = 2\ N/mm^2; \quad \rho = 0.000\,98\ kg/cm^3;}$$

$$\mathbf{\therefore w = 0.00098 \times 9.81 \times 10^{-3}\ N/mm^3 = 9.6138 \times 10^{-6}\ N/mm^3; \quad \mu = 0.35}$$

Solution :

(i) Velocity

$$v = \frac{\pi(d+t)n}{60{,}000} = \frac{\pi(900+9)336}{60{,}000} = 15.992\ \text{m/sec}$$

(ii) Centrifugal stress

$$\sigma_c = \frac{w}{g}v^2 \times 10^6 = \frac{9.6138 \times 10^{-6} \times 15.992^2 \times 10^6}{9810} = \mathbf{0.251\ N/mm^2}$$

(iii) Capacity

$$e^{\mu\theta} = e^{0.35\times 120}\times\frac{\pi}{180} = 2.08$$

(iv) Constant

$$k = \frac{e^{\mu\theta}-1}{e^{\mu\theta}} = \frac{2.08-1}{2.08} = 0.52$$

(v) Power capacity of the belt

$$\text{Power transmitted per mm}^2\text{ area} = \frac{(\sigma_1-\sigma_C)kv}{1000} = \frac{(2-0.251)0.52\times 15.992}{1000} = 0.01454\text{ kW} \quad \text{---- 21.4a}$$

$$\text{Area of cross section of belt} = b\times t = 250\times 9 = 2250\text{ mm}^2$$

$$\therefore\ \text{Power capacity } N = \frac{\text{Power}}{\text{mm}^2}\times\text{Area of cross section of belt in mm}^2$$

$$= 0.01454\times 2250 = \mathbf{32.725\ kW}$$

Example 8.7

A nylon core flat belt 200 mm wide weighing 20 N/m, connecting a 300 mm diameter pulley to a 900 mm diameter driven pulley at a shaft spacing of 6 m, transmits 55.2 kW at a belt speed of 25 m/sec (i) Calculate the belt length and the angles of wrap; (ii) Compute the belt tensions based on a coefficient of friction 0.38

VTU Mar. 2001

Data :

$$\mathbf{b = 200\ mm;\ d_1 = 300\ mm = d;\ d_2 = 900\ mm = D;\ C = 6\ m = 6000\ mm;}$$
$$\mathbf{N = 55.2\ kW;\ v = 25\ m/sec;\ \mu = 0.38;\ Weight\ per\ meter\ length = 20\ N/m}$$

Solution :

(i) Length of belt and angles of wrap

Assume open belt

$$\theta_L = \pi + \left\{2\sin^{-1}\left(\frac{D-d}{2C}\right)\right\}\frac{\pi}{180} \quad \text{---- 21.10a (DDHB)}$$

$$= \pi + \left[2\sin^{-1}\left(\frac{900-300}{2\times 6000}\right)\right]\frac{\pi}{180} = 3.2416\text{ radian}$$

$$= 185.732^\circ = \text{Angle of contact for larger pulley}$$

$$\theta_S = \pi - \left\{2\sin^{-1}\left(\frac{D-d}{2C}\right)\right\}\frac{\pi}{180} \quad \text{---- 21.10b (DDHB)}$$

$$= \pi - \left[2\sin^{-1}\left(\frac{900-300}{2\times 6000}\right)\right]\frac{\pi}{180} = 3.04155\text{ radian}$$

$$= 174.268^\circ = \text{Angle of contact for smaller pulley}$$

$$\text{Length of open belt } L = \sqrt{4C^2-(D-d)^2} + \frac{1}{2}(D\theta_L + d\theta_S)$$

$$= \sqrt{4\times6000^2-(900-600)^2}+\frac{1}{2}(900\times3.2416+300\times3.04155)$$

$$= 13895.893\,\text{mm} \approx 13.896\,\text{m}$$

(ii) Belt tensions

Since μ is same for both pulleys, capacity $e^{\mu\theta} = e^{\mu\theta_s} = e^{0.38\times3.04155} = 3.1765$

$$\text{Constant } k = \frac{e^{\mu\theta}-1}{e^{\mu\theta}} = \frac{3.1765-1}{3.1765} = 0.6852$$

$$\text{Centrifugal tension } T_C = \frac{Wv^2}{g} = \frac{20\times25^2}{9.81} = 1274.21\,\text{N}$$

$$\text{Total power transmitted by the belt } N = \frac{(T_1-T_C)kv}{1000}\,\text{kW}$$

$$\text{i.e., } 55.2 = \frac{(T_1-1274.21)0.6852\times25}{1000}$$

$$\therefore \text{Tension on tight side } T_1 = 4496.627\,\text{N}$$

$$\text{Also } \frac{T_1-T_C}{T_2-T_C} = e^{\mu\theta}$$

$$\text{i.e., } \frac{4496.627-1274.21}{T_2-1274.21} = 3.1765$$

$$\therefore \text{Tension on slack side } T_2 = 2288.665\,\text{N}$$

8.3 V-BELT DRIVE

8.3.1 Introduction

When the distance between the shafts is less, then V-belts are preferred. These are endless and of trapezoidal cross section as shown in Fig. 8.7. It consists of central layer of fabric and moulded in rubber or rubber like compound. This assembly is enclosed in an elastic wearing cover. The belt will have contact at the two sides of the groove in the pulley. The wedging action between the belt and the groove will increase the coefficient of friction making the drive a positive one.

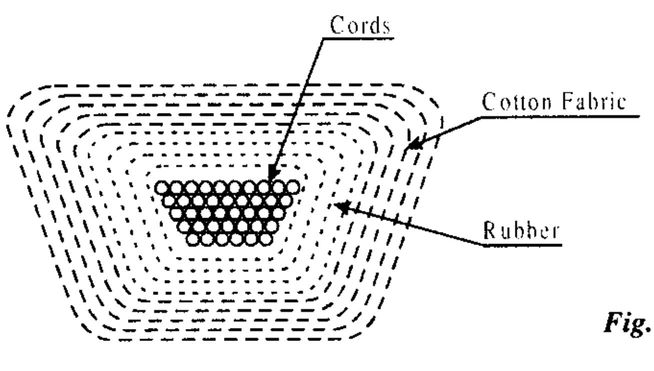


Fig. 8.7

8.3.2 Advantages of V-belt over Flat belt

Advantages :

1. Compact and give high velocity ratio.
2. Provides shock absorption between driver and driven shafts.
3. Positive and reliable drive.
4. Because of wedging action in the grooves, loss of power due to slip is less.
5. There is no joints problem as the drive is of endless.

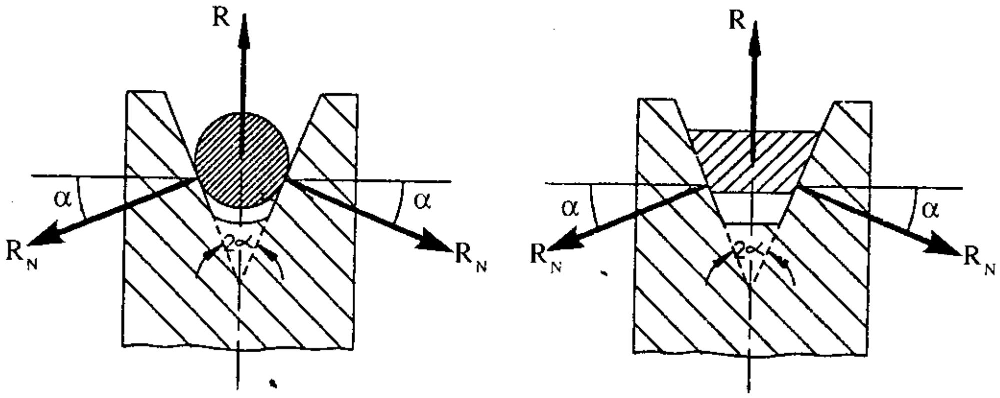
Disadvantages :

1. Initial cost is more as the fabrication of pulleys with V-grooves are complicated.
2. Cannot be used when the centre distance is large.
3. Improper belt tensioning and mismatching of belt results in reduction in service life.

8.3.3 Ratio of Belt Tensions for V-belt or Rope Drive

V-Belt drive or rope drive runs in a V-grooved pulley as discussed earlier.

The cross-section of V-belt is shown in Fig. 8.8b.



(a) Rope *(b) V-belt*

Fig.8.8

Let 2α = angle of groove

R_N = normal reaction between each side of groove and the corresponding side of the belt strip PQ

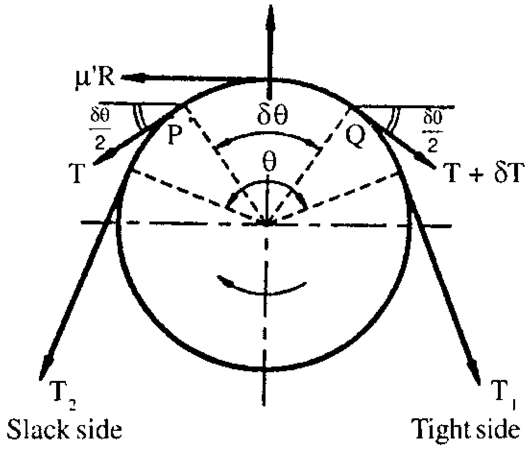
From Fig. 8.8b Resolving Forces Vertically,

$$R = R_N \sin\alpha + R_N \sin\alpha = 2 R_N \sin\alpha \quad \text{..... (i)}$$

$$\text{Total frictional force} = \mu R_N + \mu R_N = 2\mu R_N \quad \text{..... (ii)}$$

In case of V-belt or rope, there are two normal reactions as shown in Fig. 8.8, so that the radial reaction R is $2 R_N \sin \alpha$ and the total frictional force = $2 (\mu R_N) = 2 \mu R_N$

Consider a short length PQ of belt subtending angle $\delta \theta$ at the centre of the pulley as shown in Fig. 8.9



Smaller Pulley (Driven)

Fig. 8.9

Let R = Radial reaction between the belt length PQ and the pulley rim = $2R_N \sin\alpha$

R_N = Normal reaction between the belt length PQ and the pulley rim.

T = Tension on slack-side of the short strip PQ

$T + \delta T$ = Tension on tight side of short strip PQ

δT = Difference in tension due to friction between the length PQ and the surface pulley rim

μ = Coefficient of friction between the belt and pulley surface

μ' = Effective coefficient of friction = $\mu/\sin\alpha$

The strip PQ will be in equilibrium (Fig. 8.10) under the action of four forces T, $T + \delta T$, $2 \mu R_N$ and R where $2 \mu R_N$ is the frictional force which is opposing the motion.

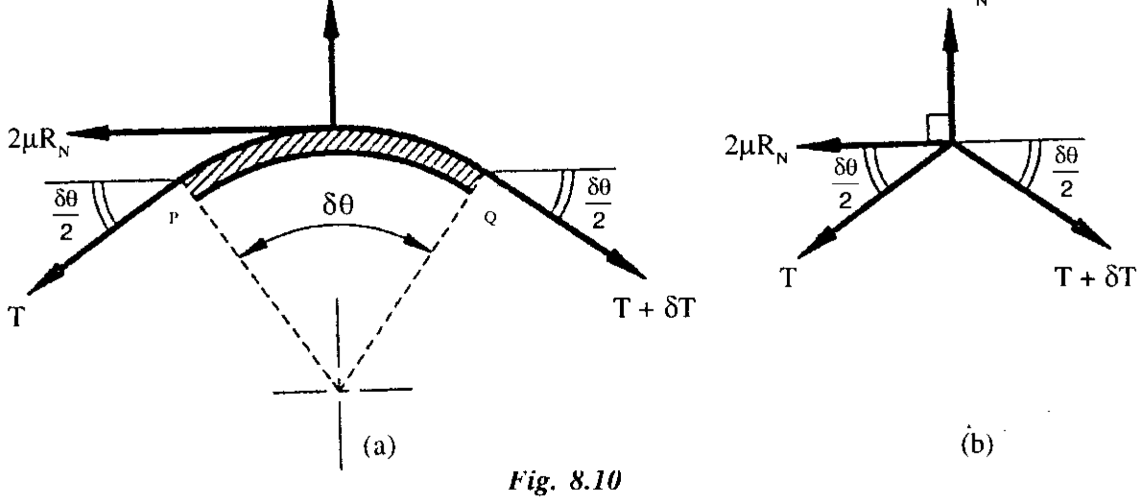


Fig. 8.10

Resolving the Forces Vertically

$$2 R_N \sin \alpha = (T + \delta T) \sin \frac{\delta\theta}{2} + T \sin \frac{\delta\theta}{2}$$

$$= T\sin\frac{\delta\theta}{2} + \delta T\sin\frac{\delta\theta}{2} + T\sin\frac{\delta\theta}{2}$$

$$= 2T\times\sin\frac{\delta\theta}{2} + \delta T\sin\frac{\delta\theta}{2}$$

$$= 2T\frac{\delta\theta}{2} + \delta T\frac{\delta\theta}{2} \qquad \text{... [As } \delta\theta \text{ is small, } \sin\frac{\delta\theta}{2} \sim \frac{\delta\theta}{2}\text{]}$$

$$= 2T\frac{\delta\theta}{2} \qquad \text{... [neglecting } \frac{\delta T.\delta\theta}{2}\text{]}$$

$$\therefore \quad 2R_N\sin\alpha = T\delta\theta$$

$$\therefore \quad R_N = \frac{1}{2\sin\alpha}T\delta\theta \qquad \text{..... (iii)}$$

Resolving the Forces Horizontally

$$2\mu R_N = (T+\delta T)\cos\frac{\delta\theta}{2} - T\cos\frac{\delta\theta}{2}$$

$$= T\cos\frac{\delta\theta}{2} + \delta T\cos\frac{\delta\theta}{2} - T\cos\frac{\delta\theta}{2}$$

$$= \delta T\cos\frac{\delta\theta}{2}$$

$$= \delta T \text{ [since } \delta\theta \text{ is small, } \cos\frac{\delta\theta}{2} \to 1\text{]}$$

$$\therefore 2\mu R_N = \delta T \qquad \text{..... (iv)}$$

Substituting value of R_N from (iii) in (iv), we get

$$2\mu\left[\frac{1}{2\sin\alpha}T\delta\theta\right] = \delta T$$

$$\text{i.e.} \quad \frac{\mu}{\sin\alpha}T\delta\theta = \delta T$$

$$\therefore \quad \frac{\delta T}{T} = \frac{\mu}{\sin\alpha}\delta\theta$$

Integrating both sides of equation between their limits

$$\int_{T_2}^{T_1}\frac{\delta T}{T} = \int_0^{\theta}\frac{\mu\delta\theta}{\sin\alpha}$$

$$\text{i.e.} \quad \left(\log_e T\right)_{T_2}^{T_1} = \frac{\mu}{\sin\alpha}(\theta)_0^{\theta}$$

$$log_e T_1 - log_e T_2 = \frac{\mu}{sin\,\alpha}\,\theta$$

$$log_e \frac{T_1}{T_2} = \frac{\mu}{sin\,\alpha}\,\theta$$

$$\therefore \frac{T_1}{T_2} = e^{\frac{\mu\theta}{sin\,\alpha}}$$ where θ in radians and α in degrees.

The above equation is called the 'limiting tension ratio' of the *V-belt or rope* and is valid only when the belt is on the point of slipping on the pulleys.

Considering centrifugal tension, ratio of belt tension = $\frac{T_1 - T_C}{T_2 - T_C} = e^{\frac{\mu\theta}{sin\,\alpha}}$

8.3.4 Design procedure for V-Belt

1. Selection of belt c/s

Equivalent pitch diameter of smaller pulley

$$d_e = d_p.F_b \qquad \text{---- 21.35 (DDHB)}$$

where $d_p = d_1$

F_b = smaller diameter factor ---- Table 21.25

Based on 'de' select the c/s of belt from page 307 (DDHB)

If 'd_1' is not given them based on *power* select the c/s of belt from table 21.24 and diameter d_1 from Table 21.31.

2. Velocity

$$v = \frac{\pi d_1 n_1}{60000} \text{ m/sec}$$

3. Power capacity

Based on the cross-section selected, calculate the power capacity N* from the formulas given in page 307 (DDHB)

4. Number of 'V' belts

$$i = \frac{N F_a}{N^* F_C . F_d} \qquad \text{---- 21.36 (DDHB)}$$

N = Total power transmitted in kW

N* = Power capacity

F_a = Service factor from Table 21.28

If the condition is not given then assume medium duty and 10-16 hours duty per day.

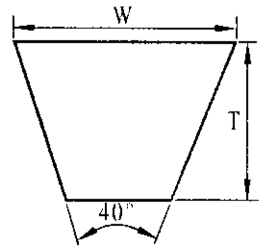
Pitch length $L = 2C + \frac{\pi}{2}(D + d) + \frac{(D-d)^2}{4C}$ ---- 21.38 (DDHB)

If centre distance 'C' is not given, then calculate

$C_{max} = 2(D + d)$ ---- 21.39 (DDHB)

$C_{min} = 0.55(D + d) + T$ ---- 21.40 (DDHB)

For 'T' from the table 21.23 for the selected cross section



Top width = W

Thickness = T

From the calculated value of C_{max} and C_{min}

Select 'C' and find 'L' Then from table 21.29 select the nearest standard value of pitch length 'L' and Nominal inside length.

Now from table 21.27 for the nominal inside length and the selected cross section, correction factor for length = F_C

$$\theta = 2\cos^{-1}\left(\frac{D-d}{2C}\right) \quad \text{---- 21.45}$$

or

$$\theta = 180 - 2\sin^{-1}\left(\frac{D-d}{2C}\right) \quad \text{---- 21.10b}$$

For open belt

From table 21.26 for the calculated value of 'θ'

Correction factor for angle of contact = F_d (If the type is not given select V – V – belt)

∴ Number of belts = i

5. Correct centre distance

$$C = \frac{L}{4} - \frac{\pi(D+d)}{8} + \sqrt{\left\{\frac{L}{4} - \frac{\pi(D+d)}{8}\right\}^2 - \frac{(D-d)^2}{8}} \quad \text{---- 21.39 (DDHB)}$$

D = larger pulley diameter

d = smaller pulley diameter

L = standard pitch length

6. Specify the V-belt by the cross section letter followed by the inside length of belt.

Example 8.8

Select a V-belt drive to transmit 10 kW of power from a pulley of 200 mm diameter mounted on an electric motor running at 720 rpm to another pulley mounted on compressor running at 200 rpm. The service is heavy duty varying from 10 hours to 14 hours per day and centre distance between centre of pulleys is 600 mm. **BU August 1999**

Data :

$$N = 10\,kW;\ d_1 = 200\ mm = d;\ n_1 = 720\ rpm;\ n_2 = 200\ rpm;\ C = 600\ mm$$

Heavy duty 10 hours to 14 hours per day.

Solution :

i. Diameter of larger pulley

$$n_1 d_1 = n_2 d_2$$

$$720 \times 200 = 200 \times d_2$$

$$\therefore\ d_2 = 720\ mm = D = \text{diameter of larger pulley}$$

ii. Select the cross-section of belt

Equivalent Pitch diameter of smaller pulley $d_e = d_p F_b$ where $d_p = d_1 = 200$ mm —21.35 (DDHB)

$$\frac{n_1}{n_2} = \frac{720}{200} = 3.6$$

From Table 21.25 when $\frac{n_1}{n_2} = 3.6$

Smaller diameter factor $F_b = 1.14$

$$\therefore\ d_e = 200 \times 1.14 = 228\ mm.$$

From page 307 (DDHB) the c/s of the belt selected corresponding to the equivalent pitch diameter $d_e = 228$ mm is 'C'

iii. Velocity

$$v = \frac{\pi d_1 n_1}{60000} = \frac{\pi \times 200 \times 720}{60000} = 7.54\ m/sec$$

iv. Power capacity

For 'C' cross-section belt

$$N^* = v\left[\frac{1.47}{V^{0.09}} - \frac{143.27}{d_e} - \frac{2.34v^2}{10^4}\right] \qquad \text{—21.32 (DDHB)}$$

$$= 7.54\left[\frac{1.47}{7.54^{0.09}} - \frac{143.27}{228} - \frac{2.34 \times 7.54^2}{10^4}\right]$$

$$N^* = \mathbf{4.4\ kW}$$

v. Number of belts

$$i = \frac{NF_a}{N^* F_c \cdot F_d} \qquad \text{—21.36 (DDHB)}$$

From Table 21.28 for heavy duty 10 – 14 hours/day correction factor for service $F_a = 1.3$

$$L = 2C + \frac{\pi}{2}(D + d) + \frac{(D - d)^2}{4C} \qquad \text{—21.38 (DDHB)}$$

$$= 2 \times 600 + \frac{\pi}{2}(720 + 200) + \frac{(720 - 200)^2}{4 \times 600} = 2757.8\ mm$$

From Table 21.29 the nearest standard value of nominal pitch length for the selected C-cross section belt $L = 2723$ mm

Nominal inside length = 2667 mm

Now from Table 21.27 for nominal inside length = 2667 mm, and C-cross-section belt, correction factor for length $F_c = 0.94$

$$\text{Angle of contact } \theta = 2\cos^{-1}\left(\frac{D-d}{2C}\right) \qquad \text{----21.45 (DDHB)}$$

$$= 2\cos^{-1}\left(\frac{720-200}{2\times 600}\right) = 128.64^\circ$$

From Table 21.26 when $\theta = 128.64^\circ$

Correction factor for angle of contact $F_d = 0.86$ (Assume V-V belt)

$$\therefore \quad i = \frac{10\times 1.3}{4.4\times 0.94\times 0.86} = 3.655$$

$\therefore$ Number of V belts **i = 4**

vi. Correct centre distance

$$C = \frac{L}{4} - \frac{\pi(D+d)}{8} + \sqrt{\left\{\frac{L}{4} - \frac{\pi(D+d)}{8}\right\}^2 - \frac{(D-d)^2}{8}} \qquad \text{----21.39 (DDHB)}$$

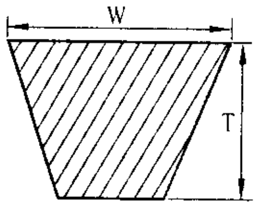
$$= \frac{2723}{4} - \frac{\pi(720+200)}{8} + \sqrt{\left\{\frac{2723}{4} - \frac{\pi(720+200)}{8}\right\}^2 - \frac{(720-200)^2}{8}}$$

$\therefore$ Correct centre distance $C = 580.73$ mm

vii. Specification of V-belt

The V-belt selected is, C 2667

$\left.\begin{array}{l} W = 22\text{ m} \\ T = 14\text{ mm} \end{array}\right\}$ From Table 21.23



Example 8.9

A compressor is driven by a motor of 2.5 kW running at 1200 rpm to a 400 rpm compressor. Select a suitable V-belt.

Data :

$$N = 2.5\text{ kW}; \quad n_1 = 1200\text{ rpm}; \quad n_2 = 400\text{ rpm}$$

Solution :

1. **Selection of Belt c/s**

   From Table 21.24 the maximum power that can be able to transmit by an 'A'-cross-section belt is 3.31 kW. Since the given power 2.5 kW is less than 3.31 kW, select *'A' Cross-section belt.*

2. **Diameter of two pulleys**

   From Table 21.31 for 'A' cross section belt pitch diameter of the smaller pulley $d_1 = 90$ mm = d [first preference is preferable]

Now
$$n_1 d_1 = n_2 d_2$$
$$1200 \times 90 = 400 \times d_2$$
$$\mathbf{d_2 = 270\ mm = D} = \text{Pitch diameter of larger pulley.}$$

3. Velocity

$$v = \frac{\pi d_1 n_1}{60000} = \frac{\pi \times 90 \times 1200}{60000} = 5.655 \text{ m/sec}$$

4. Power capacity

For 'A' cross-section belt

$$N^* = V\left(\frac{0.45}{v^{0.09}} - \frac{19.62}{d_e} - \frac{0.765 v^2}{10^4}\right) \quad \text{----21.30(DDHB)}$$

$$d_e = d_p F_b \text{ Where } d_p = d_1 = 90 \text{ mm} \quad \text{----21.35(DDHB)}$$

$$\frac{n_1}{n_2} = \frac{1200}{400} = 3$$

From Table 21.25 when $\frac{n_1}{n_2} = 3$

Smaller diameter factor $F_b = 1.14$

$\therefore\ d_e = 90 \times 1.14 = 102.6 \text{mm}$

$$\therefore\ N^* = 5.655\left[\frac{0.45}{5.655^{0.09}} - \frac{19.62}{102.6} - \frac{0.765 \times 5.655^2}{10^4}\right]$$

$$\mathbf{N^* = 1.11\ kW}$$

5. Number of V belts

$$i = \frac{N.F_a}{N^* F_c . F_d} \quad \text{----21.36(DDHB)}$$

From Table 21.28 for the driven mechanism compressor, assuming for 10 to 16 hours duty

Correction factor for service $F_a = 1.1$

$$\text{Pitch length } L = 2C + \frac{\pi}{2}(D + d) + \frac{(D - d)^2}{4C} \quad \text{----21.38(DDHB)}$$

$$C_{max} = 2(D + d) = 2[270 + 90] = 720 \text{ mm} \quad \text{----21.39(DDHB)}$$

$$C_{min} = 0.55(D + d) + T \quad \text{----21.40(DDHB)}$$

From Table 21.23 for 'A' cross-section belt

Top width $W = 13$mm

Thickness $T = 8$mm

$\therefore\ C_{min} = 0.55[270 + 90] + 8 = 206$mm

Select $\mathbf{C = 500\ mm}$ = Centre distance

$$\therefore\ L = 2 \times 500 + \frac{\pi}{2}[270 + 90] + \frac{(270 - 90)^2}{4 \times 500} = 1581.69 \text{ mm}$$

From Table 21.29 for 'A' cross section belt, the nearest standard pitch Length L = 1560 mm

Nominal inside length = *1524 mm*

From Table 21.27 when inside length = 1524 mm

Correction factor for length $F_c = 0.98$ for A-cross section belt.

$$\text{Angle of contact } \theta = 2\cos^{-1}\left(\frac{D-d}{2C}\right) = 2\cos^{-1}\left(\frac{270-90}{2\times 500}\right) = 159.26° \text{—21.45(DDHB)}$$

From table 21.26 when θ = 159.26°

Correction factor for angle F_d = 0.95 for V-V-belt [Assume V-V-belt]

$$\therefore \ i = \frac{2.5\times 1.1}{1.11\times 0.98\times 0.95} = 2.66$$

$\therefore$ Number of V-belts i = 3

6. Correct centre distance

$$C = \frac{L}{4} - \frac{\pi(D+d)}{8} + \sqrt{\left\{\frac{L}{4} - \frac{\pi(D+d)}{8}\right\}^2 - \frac{(D-d)^2}{8}} \text{ ----21.39(DDHB)}$$

$$= \frac{1560}{4} - \pi\left(\frac{270+90}{8}\right) + \sqrt{\left\{\frac{1560}{4} - \pi\left(\frac{270+90}{8}\right)\right\}^2 - \frac{(270-90)^2}{8}}$$

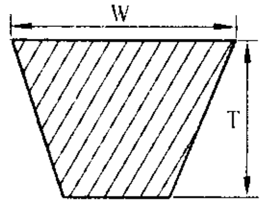
$$= \mathbf{488.974\ mm}$$

7. Specification of V-belt

The V-belt selected is A-1524

W = 13 mm

T = 8 mm



Example 8.10

A V-belt is to transmit 20 kW from a 250 mm pitch diameter sheave operating at 1500 rpm to a 900 mm diameter flat pulley. The centre distance between input and output shafts is 1 m. The groove angle is 40° and coefficient of friction is 0.2 for both pulleys and sheaves combination. The cross-section of the belt is 38 mm wide at the top and 19 mm wide at the bottom by 25 mm deep. Each belt weighs 11 kN/m³ and allowable tension per belt is 1000 N. How many belts are required.

BU September 1998

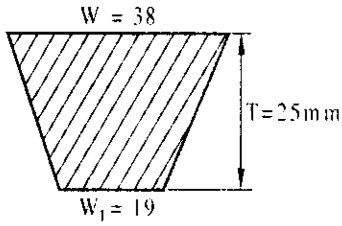
Data :

N = 20 kW; d_1 = 250 mm; n_1 = 1500 rpm; diameter of larger pulley = 900 mm

C = 1 m = 1000 mm; 2α = 40°; w = 11 kN/m³ = 11×10^{-6} N/mm³; T_1 = 1000 N.

Solution :

1. Diameter of larger pulley



Distance from the base to the centroid of the V-belt section

$$\bar{x} = \frac{T}{3}\left(\frac{W_1 + 2W}{W_1 + W}\right) = \frac{25}{3}\left[\frac{19 + 2\times38}{19+38}\right] = 13.9 \text{ mm}$$

$\therefore$ Pitch diameter of larger pulley $d_2 = 900 + 2 \times 13.9$

$$\therefore \mathbf{d_2 = 927.8\ mm = D}$$

Pitch diameter of the larger pulley is measured to the centroid of belt section, since it is a flat pulley.

2. Velocity

$$v = \frac{\pi d_1 n_1}{60000} = \frac{\pi \times 250 \times 1500}{60000} = 19.635 \text{ m/sec}$$

3. Centrifugal stress

$$\sigma_c = \frac{wv^2}{g} \times 10^6 = \frac{11\times10^{-6}}{9810} \times 19.635^2 \times 10^6 = \mathbf{0.4323\ N/mm^2} \qquad \text{----21.3b(DDHB)}$$

4. Capacity

$$\theta_s = \pi - \left\{2\sin^{-1}\left(\frac{D-d}{2C}\right)\right\}\frac{\pi}{180} \qquad \text{----21.10b(DDHB)}$$

$$= \pi - \left\{2\sin^{-1}\left(\frac{927.8-250}{2\times1000}\right)\right\}\frac{\pi}{180} = 2.45 \text{ radians.}$$

$$\theta_l = \pi + \left\{2\sin^{-1}\frac{D-d}{2C}\right\}\frac{\pi}{180} = 3.833 \text{ radians} \qquad \text{----21.10a(DDHB)}$$

Smaller pulley is the grooved pulley

$$e^{\frac{\mu\theta_s}{\sin\alpha}} = e^{\frac{0.2\times2.45}{\sin 20}} = 4.19$$

Larger pulley is the flat pulley

$$e^{\mu\theta_L} = e^{0.2\times3.833} = 2.15$$

Smaller value is the power capacity $\therefore$ Power capacity is based on larger pulley.

5. Constant 'k'

Since the capacity based on the larger pulley, $k = \frac{e^{\mu\theta}-1}{e^{\mu\theta}} = \frac{2.15-1}{2.15} = 0.535$

6. Power transmitted per belt

$$N = \frac{(T_1 - T_c)kv}{1000} \text{ kW}$$

$$T_c = \sigma_c \times \text{Area of c/s of belt}$$

Area of c/s of belt $= \frac{1}{2}$ T (W + W_1) $= \frac{1}{2} \times 25\ [38 + 19] = 712.5\ mm^2$

$$T_c = 0.4323 \times 712.5 = 308 \text{ N}$$

$$\therefore N = \frac{[1000-308]0.535\times19.635}{1000} = 7.269 \text{ kW per belt}$$

7. Number of belts

$$\text{Number of belts } i = \frac{\text{Total power}}{\text{Power transmitted / belt}} = \frac{20}{7.269} = 2.75$$

$$\therefore \text{ Number of V-belts } i = 3$$

Example 8.11

A V-belt is to be arranged between two shafts whose centres are 3000 mm. The driving pulley is of 850 mm effective diameter and is to be supplied with 75 kW at 960 rpm. The follower pulley is to run at 480 rpm. Determine the number of belts required for the following particulars.

Area of belt section = 400 mm²
Weight of belt = 0.01 N/cm³
Safe working tensile stress = 2.1 N/mm²
Coefficient of friction = 0.27
Groove angle of pulley = 40°

Also find the initial tension required in each belt.

Data :

$C = 3000$ mm; $w = 0.01$ N/cm³ $= 10 \times 10^{-6}$ N/mm³; $d_1 = 850$ mm $= d$; $N = 75$ kW;
$\sigma_t = 2.1$ N/mm²; $n_1 = 960$ rpm; $\mu = 0.27$ $n_2 = 480$ rpm; $2\alpha = 40°$ $\therefore$ $\alpha = 20°$
$A = 400$ mm²

Solution :

i. Diameter of larger pulley

$$n_1 d_1 = n_2 d_2$$
$$960 \times 850 = 480 \times d_2$$
$$\therefore \; d_2 = 1700 \text{ mm} = D$$

ii. Velocity

$$v = \frac{\pi d_1 n_1}{60000} = \frac{\pi \times 850 \times 960}{60000} = 42.726 \text{ m/sec}$$

iii. Centrifugal stress

$$\sigma_c = \frac{w}{g} v^2 \times 10^6 = \frac{10 \times 10^{-6} \times 42.726^2 \times 10^6}{9810} = 1.861 \text{ N/mm}^2 \qquad \text{----21.3b (DDHB)}$$

iv. Capacity

$$\theta_s = \pi - \left\{2\sin^{-1}\left(\frac{D-d}{2C}\right)\right\}\frac{\pi}{180} \qquad \text{----21.10b (DDHB)}$$

$$= \pi - \left\{2\sin^{-1}\left(\frac{1700-850}{2 \times 3000}\right)\right\}\frac{\pi}{180} = 2.8573 \text{ radians}$$

$$\theta_L = \pi + \left\{2\sin^{-1}\left(\frac{D-d}{2C}\right)\right\}\frac{\pi}{180} \qquad \text{----21.10a (DDHB)}$$

$$= \pi + \left\{2\sin^{-1}\left(\frac{1700-850}{2\times 3000}\right)\right\}\times\frac{\pi}{180} = 3.4259 \text{ radians}$$

$$e^{\frac{\mu\theta_S}{\sin\alpha}} = e^{\frac{0.27\times 2.8573}{\sin 20}} = 9.54$$

$$e^{\frac{\mu\theta_L}{\sin\alpha}} = e^{\frac{0.27\times 3.4259}{\sin 20}} = 14.95$$

Smaller value is the capacity

$$\therefore \text{ Capacity} = e^{\frac{\mu\theta_S}{\sin\alpha}} = 9.54$$

v. Constant

$$k = \frac{e^{\frac{\mu\theta_S}{\sin\alpha}}-1}{e^{\frac{\mu\theta_S}{\sin\alpha}}} = \frac{9.54-1}{9.54} = 0.8952$$

vi. Power transmitted per belt $= \dfrac{(T_1 - T_c)kv}{1000}$ kW

$T_1 = \sigma_1 \times$ Area of c/s of belt $= 2.1 \times 400 = 840$ N

$T_c = \sigma_c \times$ c/s of belt $= 1.861 \times 400 = 744.4$ N

$$\therefore \text{ Power/belt} = \frac{(840-744.4)(0.8952)(42.726)}{1000} = 3.656 \text{ kW}$$

vii. Number of V. belts

$$i = \frac{\text{Total power transmitted}}{\text{Power transmitted / belt}} = \frac{75}{3.656} = 20.5$$

$\therefore$ Number of V-belts $i = 21$

Initial tension

$$2\sqrt{T_0} = \sqrt{T_1} + \sqrt{T_2} \qquad \text{----21.12(DDHB)}$$

$$\frac{\sigma_1 - \sigma_c}{\sigma_2 - \sigma_c} = e^{\frac{\mu\theta}{\sin\alpha}}$$

$$\frac{2.1 - 1.861}{\sigma_2 - 1.861} = 9.54$$

$$\therefore \ \boldsymbol{\sigma_2 = 1.886 \text{ N/mm}^2}$$

$T_2 = \sigma_2 \times$ Area of c/s of belt $= 1.886 \times 400 = 754.421$ N

$$\therefore \ 2\sqrt{T_0} = \sqrt{840} + \sqrt{754.421}$$

$\therefore$ **Initial tension T_0 = 796.64 N**

8.4 ROPE DRIVES

8.4.1 Introduction

When power is to be transmitted over long distances then belts cannot be used due to the heavy losses in power. In such cases ropes can be used. Ropes are used in elevators, mine hoists, cranes, oilwell drilling, aerial conveyors, tramways, haulage devices, lifts and suspension bridges etc. Two types of ropes are commonly used. They are (i) fibre ropes (ii) metallic ropes. Fibre ropes are made of Manila, hemp, cotton, jute, nylon coir etc., and are normally used for transmitting power. Metallic ropes are made of steel, aluminium alloys, copper, bronze or stainless steel and are mainly used in elevator, mine hoists, cranes, oil-well drilling, aerial conveyors, haulage devices and suspension bridges.

8.4.2 Hoisting tackle (Block and Tackle Mechanism)

It consists of two pulley blocks one above the other. Each block has a series of sheaves mounted side by side on the same axle. The ropes used in hoisting tackle are (i) Cotton ropes (ii) Hemp ropes and (iii) Manila ropes. The pulleys are manufactured in two designs (i) fixed pulley (ii) Movable pulley.

Pulley system

A pulley system is a combination of several movable and fixed pulleys or sheaves. The system can be used for a gain in force or for a gain in speed. Hoisting devices employ pulleys for a gain in force predominantly. Pulley systems for a gain in forces are designed (i) with the rope running off a fixed pulley (ii) with the rope running off a movable pulley. Consider a hoisting tackle (block and tackle mechanism) as shown in Fig. 8.11.

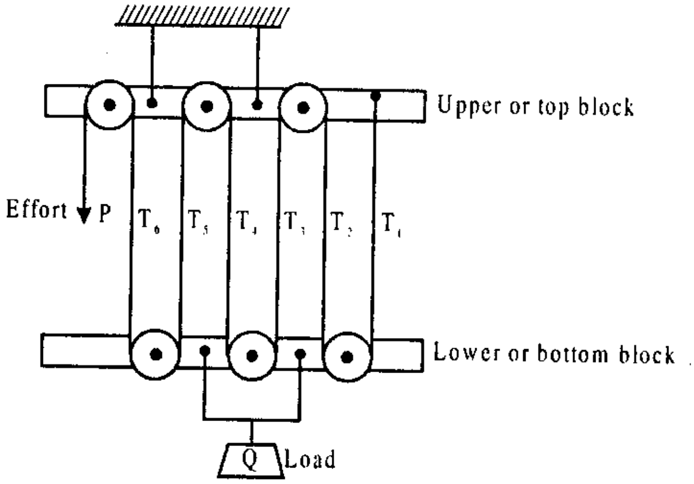


Fig.8.11 [21.7 DDHB]

Let Q = Load to be lifted

P = Effort required to raise the load

n = Total number of pulleys or sheaves

T_1, T_2, T_3, ---- etc. = Tensions in the rope

C = Pulley coefficient or Ratio of adjacent tensions.

i) Raise the load

For raising the load the block is pulled downward at P

$$C = \frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3} = \frac{T_5}{T_4} = \frac{T_6}{T_5} = \frac{P}{T_6}$$

$$\therefore\ T_2 = CT_1$$
$$T_3 = CT_2 = C^2 T_1$$
$$T_4 = CT_3 = C^3 T_1$$
$$T_5 = CT_4 = C^4 T_1$$
$$T_6 = CT_5 = C^5 T_1$$
$$P = CT_6 = C^6 T_1$$

For equilibrium at the lower block

$$Q = T_1 + T_2 + T_3 + T_4 + T_5 + T_6$$
$$= T_1 + CT_1 + C^2 T_1 + C^3 T_1 + C^4 T_1 + C^5 T_1$$
$$= T_1 [1 + C + C^2 + C^3 + C^4 + C^5]$$

$$\therefore\ Q = T_1 \left[\frac{C^6 - 1}{C - 1}\right] \quad (\because\ 1 + C + C^2 + \text{-----} \text{ is the geometric progression})$$

$$= \frac{P}{C^6}\left(\frac{C^6 - 1}{C - 1}\right) \quad (\because\ P = C^6 T_1]$$

$\therefore$ Effort required to raise the load $\mathbf{P = \dfrac{QC^6 (C-1)}{(C^6 - 1)}}$

For 'n' number of pulleys or sheaves $\mathbf{P = \dfrac{QC^n (C-1)}{(C^n - 1)}}$ ---- 21.60 (DDHB)

ii) Lowering the load

For lowering the load the block is pulled up at 'P'

$$\text{Hence } C = \frac{T_1}{T_2} = \frac{T_2}{T_3} = \frac{T_3}{T_4} = \frac{T_4}{T_5} = \frac{T_5}{T_6} = \frac{T_6}{P}$$

$$T_6 = CP$$
$$T_5 = CT_6 = C^2 P$$
$$T_4 = CT_5 = C^3 P$$
$$T_3 = CT_4 = C^4 P$$
$$T_2 = CT_3 = C^5 P$$
$$T_1 = CT_2 = C^6 P$$

From equilibrium at the lower block,

$$\begin{aligned} Q &= T_6 + T_5 + T_4 + T_3 + T_2 + T_1 \\ &= CP + C^2P + C^3P + C^4P + C^5P + C^6P \\ &= CP\,[1 + C + C^2 + C^3 + C^4 + C^5] \\ &= CP\left[\frac{C^6 - 1}{C - 1}\right] \end{aligned}$$

$\therefore$ Effort required to lower the load $\mathbf{P = \frac{Q}{C}\left[\frac{C-1}{C^6-1}\right]}$

For 'n' number of pulleys or sheaves $P = \frac{Q}{C}\left[\frac{C-1}{C^n-1}\right]$ ---- 21.61 (DDHB)

iii) Efficiency

If there is no frictional loss i.e., no friction then C = 1

$$\begin{aligned} \therefore\ Q &= T_1 + T_2 + T_3 + T_4 + T_5 + T_6 \\ &= 6T_1 \quad [\because\ T_1 = T_2 = T_3 = T_4 = T_5 = T_6] \\ &= 6P_o \quad (\because\ T_1 = P_o) \end{aligned}$$

$$\therefore\ \text{Effect } P_o = \frac{Q}{6}$$

For 'n' number of pulleys $P_o = \frac{Q}{n}$

$$\text{Efficiency } \eta = \frac{P_o}{P} = \frac{\frac{Q}{n}}{\frac{QC^n(C-1)}{(C^n-1)}}$$

$$\therefore\ \eta = \left[\frac{(C^n-1)}{nC^n(C-1)}\right]$$ ---- 21.62 (DDHB)

C From table 21.34

Example 8.12

In a block and tackle mechanism, four sheaves at the top block and four at the bottom block. Derive the expression for the effort 'P' required to raise the load interms of Q and pulley coefficient 'C'. If the load Q is 30 kN, find the effort required to raise the load, lower the load and efficiency. Take C = 1.12.

Data :

$$\mathbf{Q = 30\ kN = 30000\ N;\quad C = 1.12;\quad \text{Total number of sheaves} = 4 + 4 = 8}$$

Solution :

a. **Derivation similar to the previous one but instead of six total 8 pulleys are used.**

b. (i) Effort required to raise the load

$$P = \frac{C^n(C-1).Q}{C^n - 1} = \frac{1.12^8[1.12-1]30000}{\left(1.12^8-1\right)} = 6038.9\text{ N} \qquad \text{---- } 21.60$$

(ii) Effort required to lower the load $P = \frac{Q}{C}\left(\frac{C-1}{C^n-1}\right) = \frac{30000}{1.12}\frac{(1.12-1)}{\left(1.12^8-1\right)} = 2177.75\text{ N}$ ---- 21.61

(iii) Efficiency $\eta = \frac{C^n-1}{nC^n(C-1)} = \frac{1.12^8-1}{8\times 1.12^8(1.12-1)} = 0.62096 = \mathbf{62.1\%}$ ---- 21.62

8.4.3 Steel wire Ropes

A wire rope is made up of strands and a strand is made up of one or more layers of wires as shown in Fig. 8.12. The number of strands in a rope denotes the number of groups of wires that are laid over the central core. For example a 6 × 19 construction means that the rope has 6 strands and each strand is composed of 19 (12/6/1) wires. The central part of the wire rope is called the core and may be of fibre, wire, plastic, paper or asbestos. The fibre core is very flexible and very suitable for all conditions.

The points to be considered while selecting a wire rope are (i) strength (ii) abrasion resistance (iii) flexibility (iv) resistance to crushing (v) fatigue strength (vi) corrosion resistance. Ropes having wire core are stronger than those having fibre core. Flexibility in rope is more desirable when the number of bends in the rope are too many.

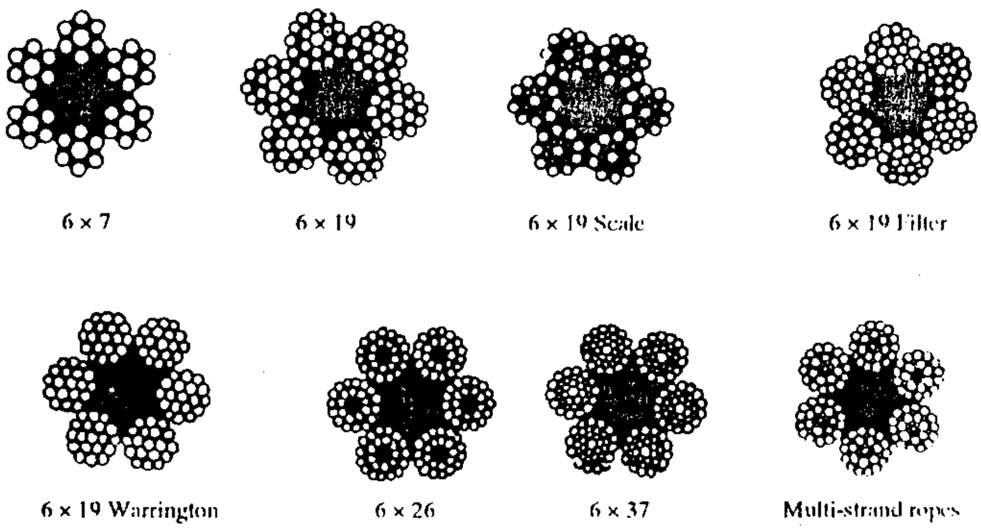


Fig. 8.12 : Steel wire rope construction

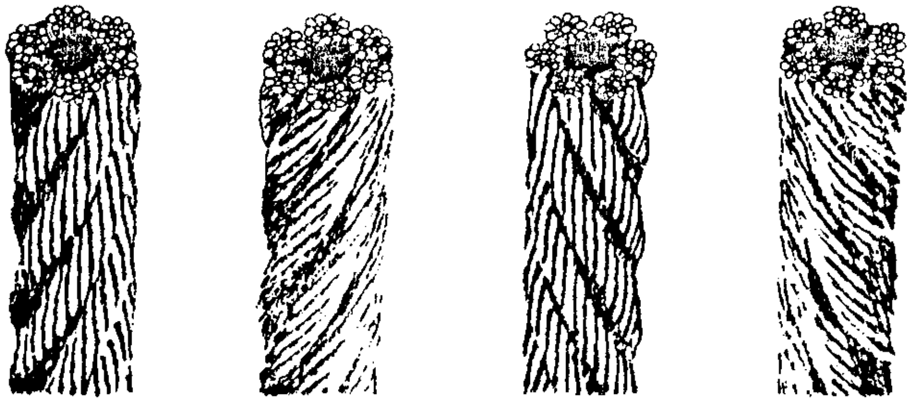


Fig. 8.13 : Rope constructions

8.4.4 Design procedure for wire rope

Let

d = Diameter of rope

D = Diameter of sheave

H = Depth of mine or height of building

W = Total load

W_R = Weight of rope

d_w = Diameter of wire

A = Area of c/s of rope

P_b = Bending load in the rope

F_a = Allowable pull in the rope

F_u = Ultimate or Breaking load of rope

n = Factor of safety. From Table 21.48

W_s = Starting load

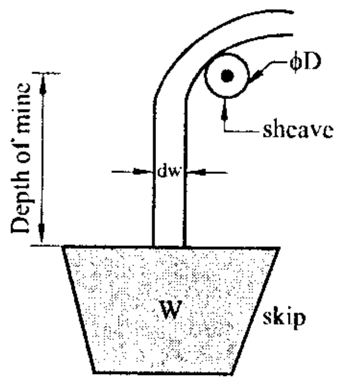


Fig. 8.14

1. ***Total load***

$$W = \text{Load to be lifted} + \text{Weight of skip}$$

2. ***Total weight of rope***

$$W_R = \text{Weight per meter length} \times \text{Length of rope}$$
$$= \text{Weight per meter length} \times H$$

3. ***Inertia load due to Acceleration***

$$W_I = Ma = \left(\frac{W + W_R}{g}\right).a$$

4. ***Bending load***

$$P_b = \frac{k.A\,d_w}{D} \qquad \text{---- 21.63 b (DDHB)}$$

Where k = 82728.5 MPa = Modulus of elasticity of rope in N/mm²

A = Area of c/s of rope in mm²

d_w = Diameter of wire in mm

D = Diameter sheave in mm

5. ***Starting load***

$$W_s = 2\,[W + W_R]$$

6. ***Maximum load***

The maximum load on the rope can be determined into the following ways.

(i) Load on the rope during uniform velocity = $W + W_R + P_b$

(ii) Load on the rope during acceleration = $W + W_R + W_I + P_b$

(iii) Load on the rope during starting = $W_s + P_b$

F_{max}, is the maximum among the above three values.

Neglecting impact $F_{max} = W + W_R + W_I + P_b$

7. ***Diameter of rope***

Allowable pull $F_a \geq F_{max}$

Since $F_a = \dfrac{F_u}{n}$

$$\frac{F_u}{n} \geq F_{max}$$

Neglecting impact,

$$\mathbf{\frac{F_u}{n} \geq W + W_R + W_I + P_b}$$

From Table 21.47, it is found that the most commonly used type of rope is 6 × 19. From Table 21.46 for 6 × 19 rope

$$F_u = 500.8\ d^2\ MN \text{ where d in meters} = 500.8\ d^2\ N \text{ where d in mm}$$

Weight per unit length of rope = $36.3\ d^2$ kN/m = $36.3\ d^2 \times 10^{-3}$ N/m where d in mm

Wire diameter $d_w = 0.063$ d mm

Area of c/s of rope $A = 0.38\ d^2\ mm^2$, d in mm

Average sheave diameter $D = 45$ d in mm

To find the acceleration any one of the following equations may be used

$$v = u + at \quad \text{---- (i)}$$

$$s = ut + \frac{1}{2}at^2 \quad \text{---- (ii)}$$

$$v^2 = u^2 + 2\ as \quad \text{---- (iii)}$$

Example 8.13

Select a wire rope to lift a load of 10 kN through a height of 600 m from a mine. The weight of bucket is 2.5 kN. The load should attain a maximum speed of 50 m/min in 2 seconds.

Solution :

From Table 21.47 select the most commonly used type of rope i.e., 6 × 19

From Table 21.46 for 6 × 19 rope $F_u = 500.8\ d^2$ N where d in mm

Weight per meter length = $36.3 \times 10^{-3}\ d^2$ N/m where d in mm

Wire diameter dw = 0.063 d, mm

Area of c/s $A = 0.38\ d^2,\ mm^2$

Sheave diameter D = 45 d, mm

From Table 21.48 for 600 m depth

F.O.S = n = 7

1. **Total load**

$$W = \text{Load to be lifted + weight of skip} = 10000 + 2500 = 12500\ N$$

2. **Total weight of rope**

$$W_R = \text{Weight per meter length} \times \text{length of rope}$$

$$= 36.3 \times 10^{-3}\ d^2 \times 600 \frac{N}{m} \text{ where d in mm } = 21.78\ d^2$$

3. **Inertia load due to Acceleration**

$$W_I = Ma = \left(\frac{W + W_R}{g}\right).a$$

$v = u + at$ Since $u = 0$

$$v = at$$

$$\therefore\ a = \frac{v}{t} = \frac{50}{60 \times 2} = 0.417\ m/sec^2$$

$$\therefore\ W_I = \left(\frac{12500 + 21.78d^2}{9.81}\right) \times 0.417 = \mathbf{531.345 + 0.9258\ d^2}$$

4. Bending load

$$P_b = k.A\frac{d_w}{D} \qquad \text{---- 21.63 b (DDHB)}$$

$$\text{where } k = 82728.5 \text{ MPa}$$

$$\therefore P_b = 82728.5 \times 0.38\,d^2 \times \frac{0.063d}{45d} = \mathbf{44.01\,d^2}$$

5. Starting load

$$W_s = 2\,[W + W_R] = 2\,[12500 + 21.78\,d^2] = 25000 + 43.56\,d^2$$

6. Maximum load

Assume impact load is neglected.

$$\therefore \text{ Max. load on the rope } F_{max} = W + W_R + W_I + P_b$$

$$\therefore F_{max} = 12500 + 21.78\,d^2 + 531.345 + 0.9258\,d^2 + 44.01\,d^2$$

$$= 13031.345 + 66.7158\,d^2$$

7. Diameter of rope

$$F_a \geq F_{max}$$

$$\text{i.e., } \frac{F_u}{n} \geq F_{max}$$

$$\therefore \frac{500.8d^2}{7} \geq 13031.345 + 66.7158\,d^2$$

$$d \geq 51.96 \text{ mm}$$

From Table 21.40 for 6 × 19 rope std diameter d = 54 mm

Example 8.14

Select a wire rope for a vertical mine hoisting to lift 12000 kN of ore in 8 hour shift from a depth of 720 m. Assume two compartment skip with the hoisting skips in balance. The maximum velocity of the rope is 750 m/min with an acceleration and deacceleration periods of 12 seconds. The rest period for load and unload is 10 seconds. The hoisting skip weights approximately 50% of the load capacity.

BU March 2000

Data :

Total load to be lifted in eight hours = 12000 kN = 12 × 10^6 N
Available time = 8 hours = 8 × 3600 = 28800 seconds. Two compartment skip
∴ Load is available in each travel. Velocity $v = 750 \text{ m/min} = \frac{750}{60} = 12.5 \text{ m/sec}$; depth H = 720 m Period of acceleration t_a = 12 seconds; Period of deacceleration t_d = 12 seconds; load and unload time t_l = 10 seconds; weight of skip = 0.5 × load capacity.

Solution :

From Table 21.48 when depth = 720 m. Factor of safety (FOS) = 6

Time required for each trip = Acceleration time + constant velocity movement time + deacceleration time + load and unload time

$$v = u + at \quad \text{Since } u = 0$$

$\therefore$ Acceleration $a = \dfrac{v}{t} = \dfrac{12.5}{12} = 1.0417 \text{ m/sec}^2$

Also $v^2 = u^2 + 2as$

$12.5^2 = 0 + 2 \times 1.0417 \times s_a$

$\therefore$ Distance moved during the period of acceleration $s_a = 74.99 \approx 75$ m

Distance moved during the period of deacceleration $s_d = 75$ m. ($\therefore$ Deacceleration time also equal to 12 seconds)

Remaining distance will be moved by constant velocity $s_c = 720 - s_a - s_d = 720 - 75 - 75 = 570$ m

$\therefore$ Time required to cover this distance with constant velocity $t_c = \dfrac{s_c}{v} = \dfrac{570}{12.5} = 45.6$ seconds

$\therefore$ Total time taken for each trip $t = t_a + t_c + t_d + t_i = 12 + 45.6 + 12 + 10 = 79.6$ seconds

$\therefore$ Number of possible trips in 8 hours $= \dfrac{8 \times 60 \times 60}{79.6} = 361.81 \approx 362$ trips.

$\therefore$ Load on each trip $= \dfrac{\text{Total load}}{\text{Number of trips}} = \dfrac{12 \times 10^6}{362} = 33149.2 \text{ N} \approx 33150 \text{ N}$

Weight skip $= 0.5 \times$ load capacity $= 0.5 \times 33150 = 16575$ N

Select the most commonly used type rope i.e., 6×19 rope

$\therefore$ From Table 21.46 for 6×19 rope, $F_u = 500.8\, d^2$, N where d in mm

Weight per meter length $= 36.3\, d^2$ kN/m $= 36.3 \times 10^{-3}\, d^2$ N/m where d in mm

Wire diameter $d_w = 0.063\, d$, mm; Area of c/s $A = 0.38\, d^2$, mm^2

Average sheave diameter $D = 45\, d$, mm

1. Total load

W = Load to be lifted per trip + weight of skip

$= 33150 + 16575 = 49725$ N

2. Total weight of rope

W_R = Weight per meter length $\times$ length of rope $= 36.3 \times 10^{-3}\, d^2 \times 720$

$= \mathbf{26.136\, d^2}$

3. Inertia load due to acceleration

$$W_I = \left(\frac{W + W_R}{g}\right).a = \left(\frac{49725 + 26.136 d^2}{9.81}\right) \times 1.0417$$

$= \mathbf{5280.18 + 2.7753\, d^2}$

4. Bending load

$$P_b = \frac{k.A\,d_w}{D} \quad \text{where } k = 82728.5 \text{ MPa} \qquad \text{---- 21.63 b (DDHB)}$$

$$= \frac{82728.5 \times 0.38 d^2 \times 0.063 d}{45 d}$$

$= \mathbf{44.01156\, d^2}$

5. Starting load

$$W_s = 2[W + W_R] = 2[49725 + 26.136 d^2] = 99450 + 52.272 d^2$$

6. Maximum load

(i) Load during uniform velocity $= W + W_R + P_b$

$$= 49725 + 26.136 d^2 + 44.01156 d^2 = 49725 + 70.14756 d^2$$

(ii) Load during acceleration $= W + W_R + W_t + p_b$

$$= 49725 + 26.136 d^2 + 5280.18 + 2.7753 d^2 + 44.01156 d^2$$

$$= 55005.18 + 72.92276 d^2$$

(iii) Load during starting $= P_b + W_s$

$$= 44.01156 d^2 + 99450 + 52.272 d^2 = 99450 + 96.28356 d^2$$

Neglecting impact

$$F_{max} = 55005.18 + 72.92276 d^2$$

7. Diameter of rope

$$F_u \geq F_{max}$$

$$\text{i.e.,} \quad \frac{F_u}{n} \geq F_{max}$$

$$\frac{500.8 d^2}{6} \geq 55005.18 + 72.92276\, d^2$$

$$\therefore \quad d \geq 72.23 \text{ mm}$$

take $\mathbf{d = 73\ mm}$

Example 8.15

A 20 mm 8 × 19 steel wire rope is used with a hoisting drum of 1 m diameter to lift a load of 20 kN. The depth of mine is 800 m and the acceleration is 3 m/sec². Determine the number of ropes required using a factor of safety 5. Neglect the weight of skip.

Data :

d = 20 mm; Type of rope = 8 × 19; D = 1 m = 1000 mm; Load to be lifted = 20 kN = 20000 N;

H = 800 m; a = 3 m/sec²; n = 5

Solution :

From Table 21.46 for 8 × 19 rope $F_u = 431.3 d^2 = 431.3 \times 20^2 = 172520$ N

Weight per meter length $= 34.3 \times 10^{-3} d^2 = 34.3 \times 10^{-3} \times 20^2 = 13.72$ N/m

Wire diameter $d_w = 0.05 d = 0.05 \times 20 = 1$ mm

Area of c/s $A = 0.35 d^2 = 0.35 \times 20^2 = 140 \text{ mm}^2$

Average sheave diameter $D = 1000$ mm (given)

1. Total load

$$W = 20000 \text{ N} \quad (\because \text{Weight of skip is neglected})$$

2. Total weight of rope

$$W_R = \text{Weight meter length} \times \text{length of rope} \times \text{number of ropes}$$

$$= 13.72 \times 800 \times i = 10976\, i$$

3. Inertia load due to acceleration

$$W_I = Ma = \frac{(W + W_R)}{g} a$$

$$= \frac{(20000 + 10976i)}{9.81} \times 3$$

$$= 6116.21 + 3356.575\ i$$

4. Bending load

$$P_b = ikA \frac{d_w}{D} \qquad \text{---- 21.63 b (DDHB)}$$

where k = Modulus of elasticity of rope = 82728.5 MPa

i = Number of ropes

$$\therefore\ P_b = i \times 82728.5 \times 140 \times \frac{1}{1000} = 11581.99\ i$$

5. Maximum load

Neglecting impact, $F_{max} = W + W_R + W_I + P_b$

$$= 20000 + 10976\ i + 6116.21 + 3356.575\ i + 11581.99\ i$$

$$= 26116.21 + 25914.565\ i$$

6. Number of ropes

$$F_a \geq F_{max}$$

$$\text{i.e., } i\frac{F_u}{n} \geq F_{max}$$

$$\therefore\ i \times \frac{172520}{5} \geq 26116.21 + 25914.565\ i$$

$$\therefore\ i \geq 3.04$$

$\therefore$ Number of ropes **i = 4**

Example 8.16

Select a wire rope of strand construction 6 × 19 to lift a load of 10 kN, through a height of 600 m required for a mine. The load should attain a speed of 5 m/sec while it travels a distance of 10 m. The weight of skip is 2.5 kN

Solution :

From Table 21.46 for 6 × 19 rope construction $F_u = 500.8\ d^2$ N where d in mm

Weight per meter length = $36.3 \times 10^{-3}\ d^2$ N/m where d in mm

Wire diameter d_w = 0.063 d, mm; Area of cross section $A = 0.38\ d^2$, mm^2

Sheave diameter D = 45 d, mm

From Table 21.48 for 600 m depth FOS = 7 = n; $v^2 = u^2 + 2as$ i.e., $5^2 = 0 + 2a \times 10$

$\therefore$ $a = 1.25$ m/sec^2. Now the solution is similar to Example 8.13.

Example 8.17

Select a 6 × 19 steel rope to lift 15kN of debris from a tunnel 200m deep. The bucket weighs 8kN. The velocity of the rope is 100m/min to be attained in 20 seconds. What will be the maximum load on the rope when there is a slack of 10m in the rope?

(VTU July/Aug. 2005)

Data:

6 × 19 steel rope; Load to be lifted = 15kN = 15×10^3 N

H = 200m; Weight of bucket = 8 kN = 8000N;

$$v = 100 \text{ m/min} = \frac{100}{60} = 1.667 \text{m/sec}; \; t = 20 \text{ seconds}$$

Slack h = 10m

Solution :

From Table 21.46 for 6 × 19 rope $F_u = 500.8d^2$ N where d in mm

$$\text{Weight per meter length} = 36.3 \times 10^{-3} d^2 \text{ N/m where d in mm}$$

$$\text{Wire diameter } d_w = 0.063d, \text{mm}$$

$$\text{Area of cross section } A = 0.38d^2, \text{mm}^2$$

$$\text{Average sheave diameter } D = 45d, \text{mm}$$

From Table 21.48 for 200m depth, select

$$\text{FOS } n = 7$$

$$v = u + at$$

$$\text{ie, } 1.667 = 0 + a \times 20$$

$$\therefore \text{Acceleration } a = 0.0833 \text{ m/sec}^2$$

1. Total load

$$W = \text{Load to be lifted + Weight of the bucket}$$

$$= 15000 + 8000 = 23000 \text{ N}$$

2. Total weight of rope

$$W_R = \text{weight per meter length} \times \text{length of rope}$$

$$= (36.3 \times 10^{-3} d^2)(200 + 10) = 7.623d^2$$

3. Inertia load due to acceleration

$$W_i = M.a = \frac{(W + W_R)}{g}.a$$

$$= \left(\frac{23000 + 7.623d^2}{9.81}\right)(0.0833)$$

$$= 195.3 + 0.06473\, d^2$$

4. Bending load

$$P_b = k.A.\frac{d_w}{D} \text{ where } k = 82728.5 \text{ MPa} \qquad \text{---- 21.63 b (DDHB)}$$

$$= \frac{(82728.5)(0.38d^2)(0.063d)}{45d} = 44.01156\ d^2$$

5. Starting load

Neglecting slack

$$W_S = 2(W + W_R)$$

$$= 2[23000 + 7.623\ d^2] = 46000 + 15.246\ d^2$$

Considering slack $h = 10m$

$$W_S = (W + W_R)\left[1 + \sqrt{1 + \frac{V_S^2 k}{\sigma l g}}\right] = (W + W_R)\left[1 + \sqrt{1 + \frac{2ahk}{\sigma l g}}\right]$$

where V_S = Velocity of rope at the instant when the rope is taut = $\sqrt{2ah}$

h = Rope slackness = 10m

a = Acceleration = 0.0833 m/sec^2

l = Length of rope = H + h = 200 + 10 = 210m

σ = Static stress in the rope = $\frac{W + W_R}{A}$

$$= \frac{(23000 + 7.623d^2)}{0.38d^2} = \frac{60526.32}{d^2} + 20.06$$

$$= \frac{60526.32 + 20.06d^2}{d^2}$$

$$\therefore W_S = (W + W_R)\left[1 + \sqrt{1 + \frac{2ahk}{\sigma l g}}\right]$$

$$= (23000 + 7.623\ d^2)\left[1 + \sqrt{1 + \frac{2 \times 0.0833 \times 10 \times 82728.5 \times d^2}{(60526.32 + 20.06d^2)(210)(9.81)}}\right]$$

$$= (23000 + 7.623\ d^2)\left[1 + \sqrt{1 + \frac{66.9d^2}{(60526.32 + 20.06d^2)}}\right]$$

6. Maximum load

i) Load during Uniform velocity $= W + W_R + P_b$

$$= 23000 + 7.623\,d^2 + 44.01156\,d^2$$

$$= 23000 + 51.63456\,d^2$$

ii) Load during acceleration $= W + W_R + W_I + P_b$

$$= 23000 + 7.623\,d^2 + 195.3 + 0.06473\,d^2 + 44.01156\,d^2$$

$$= 23195.3 + 51.7\,d^2$$

iii) Load during starting $= W_S + P_b$

Neglecting slack

Load during starting $= 46000 + 15.246\,d^2 + 44.01156\,d^2$

$$= 46000 + 59.25756\,d^2$$

Considering slack

$$\text{Load during starting} = (23000 + 7.623\,d^2)\left[1 + \sqrt{1 + \frac{66.9\,d^2}{60526.32 + 20.06\,d^2}}\right] + 44.01156\,d^2$$

7. Diameter of rope

For safer design

$$F_a \geq F_{max}$$

$$\text{i.e., } \frac{F_u}{n} = F_{max}$$

Neglecting slack

$$F_{max} = 46000 + 59.25756\,d^2$$

$$\text{i.e., } \frac{500.8\,d^2}{7} \geq 46000 + 59.25756\,d^2$$

$$\therefore d \geq 61.2\,\text{mm}$$

Considering slack, take diameter of rope d = 70mm

8. Maximum load

i) During uniform velocity $F_{max} = 23000 + 51.63456\,d^2$

$$= 23000 + 51.63456 \times 70^2 = 276009.344\,\text{N}$$

ii) During acceleration $F_{max} = 23195.3 + 51.7\,d^2$

$$= 23195.3 + 51.7 \times 70^2 = 276525.3\,\text{N}$$

iii) Considering slack during starting

$$F_{max} = (23000 + 7.623\,d^2)\left[1 + \sqrt{1 + \frac{66.9\,d^2}{60526.32 + 20.06\,d^2}}\right] + 44.01156\,d^2$$

$$= (23000 + 7.623 \times 70^2)$$

$$\left[1+\sqrt{1+\frac{66.9\times70^2}{60526.32+20.06\times70^2}}\right]+44.01156\times70^2$$

$$= 381653.124\,\text{N}$$

9. Checking

During uniform velocity, FOS $n = \dfrac{500.8\,d^2}{F_{max}} = \dfrac{500.8\times70^2}{276009.344} = 8.89$

During acceleration, FOS $n = \dfrac{500.8\times70^2}{276525.3} = 8.874$

During starting with slack, FOS $n = \dfrac{500.8\times70^2}{381653.124} = 6.43$

Since 6.43 is nearer to 7, it is satisfactory

Hence, diameter of rope $d = 70\text{mm}$

Maximum load with 10m slack $F_{max} = 381653.124\,\text{N} = 381.653\,\text{kN}$

Example 8.18

Select a suitable wire rope to raise a load of 10kN through a height of 400m The desired velocity of 20m/min is to be achieved while traversing through a distance of 10m from the start.

(VTU Jan/Feb2005, Jan/Feb 2006)

Solution :

From Table 21.47 select the most commonly used type of rope i.e., 6 × 19

From Table 21.46 for 6 × 19 rope

$F_u = 500.8d^2$ MN where d in meters

$= 500.8\,d^2$ N where d in mm

Weight per unit length of rope $= 36.3d^2$ kN/m

$= 36.3d^2 \times 10^{-3}$ N/m where d is mm

Wire diameter $d_w = 0.063d$, mm where d in mm

Area of c/s of rope $A = 0.38d^2$, mm² where d in mm

Average sheave diameter $D = 45d$, mm where d in mm

From Table 21.48 for 400m, factor of safety n = 7

We know

$$v^2 = u^2 + 2as$$

$$\left(\frac{20}{60}\right)^2 = 0 + 2a\times10\ (\because v = 20\text{m/min} = \frac{20}{60}\ \text{m/sec})$$

$\therefore$ Acceleration $a = 5.5556 \times 10^{-3}$ m/sec^2

Now the solution is similar to Example 8.13.

8.5 CHAIN DRIVE

8.5.1 Introduction

Chain is used to transmit motion from one shaft to another shaft with the help of sprockets. Chain drives maintain a positive speed ratio between driving and driven components, so tension on the slack side is considered as zero. They are generally used for the transmission of power in cycles, motor vehicles, agricultural machinery, road rollers etc.

8.5.2 Merits and demerits of chain drives

Merits

1. Chain drives are positive drives and can have high efficiency when operating under ideal conditions.
2. It can be used for both relatively long or short centre distances.
3. Less load on shafts and compact in size as compared to flat belt drive.

Demerits

1. Relatively high production cost and noisy operation.
2. Chain drives require more amount of servicing and maintenance as compared to belt drives.

Velocity ratio in chain drive

Let n_1 = speed of driver sprocket in rpm

n_2 = speed of driven sprocket in rpm

z_1 = number of teeth on driver sprocket

z_2 = number of teeth on driven sprocket

$$\therefore \text{ Velocity ratio } \frac{n_1}{n_2} = \frac{z_2}{z_1}$$

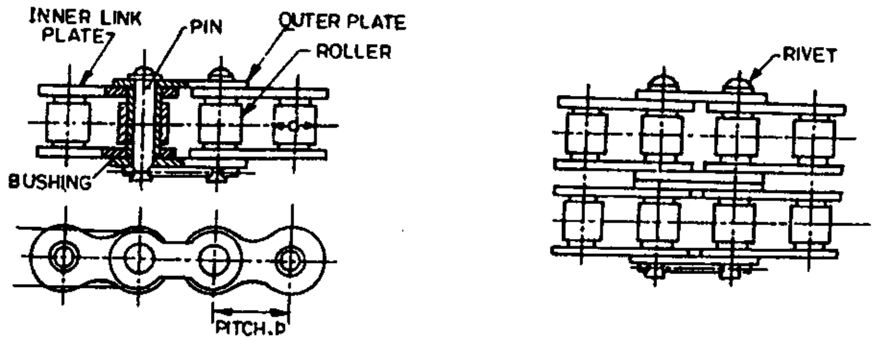
8.5.3 Chains for power transmission

The different types of chain used for power transmission are:

(i) Block chain (ii) Roller chain (iii) Inverted-tooth chain or silent chain

8.5.4 Roller chain

It consists of two rows of outer and inner plates. The outer row of plates is known as pin link or coupling link whereas the inner row of plates is called roller link. A pin passes through the bush which is secured in the holes of the inner pair of links and is riveted to the outer pair of links as shown in Fig. 8.15. Each bush is surrounded by a roller. The rollers run freely on the bushes and the bushes turn freely on the pins.



(a) (b)

Fig. 8.15

A roller chain is extremely strong and simple in construction. It gives good service under severe conditions. To avoid longer sprocket diameter, multi-row-roller chains or chains with multiple strand width are used. Theoretically, the power capacity of multistrand chain is equal to the capacity of the single chain multiplied by the number of strands, but actually it is reduced by 10 percent.

8.5.5 Inverted tooth chain or silent chain

It is as shown in Fig. 8.16. These chains are not exactly silent but these are much smoother and quieter in action than a roller chain. These chains are made up of flat steel stamping, which makes it easy to built up any width desired. The links are so shaped that they engage directly with sprocket teeth. In design, the silent chains are more complex than bush roller types, more expensive and require more careful maintenance.

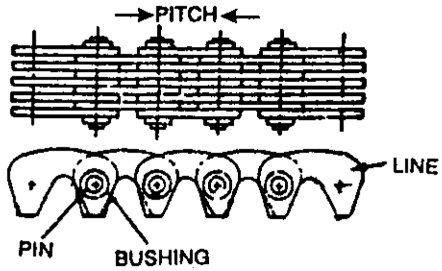


Fig. 8.16

8.5.6 Chordal action

When a chain passes over a sprocket, it moves as a series of chords instead of a continuous arc as in the case of a belt drive. Thus the centre line of a chain is not at a uniform radius. When the driving sprocket moves at a constant speed, the driven sprocket rotates at a varying speed due to the continually varying radius of chain line. This variation in speed ranges from

$$v_{min} = \frac{\pi d_1 n_1}{60 \times 1000} \times \frac{180}{z_1} \text{ to } v_{max} = \frac{\pi d_1 n_1}{60 \times 1000} \text{ m/sec}$$

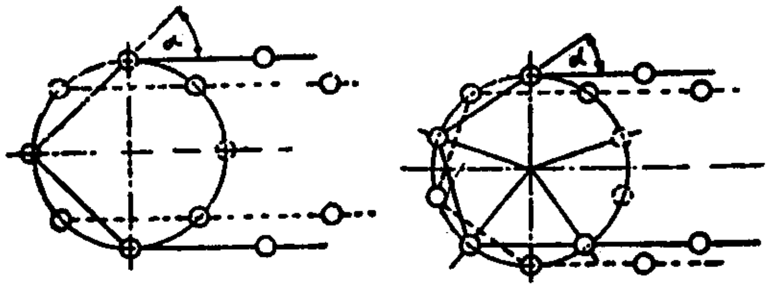
Where n_1 = Speed of the driving sprocket in rpm

d_1 = Pitch circle diameter of the driving sprocket in mm

z_1 = Number of teeth on driving sprockets.

It is clear from above that for the same pitch, the variation in speed and articulation angle α decreases, if the number of teeth in sprocket is increased. The average speed of the sprocket is given by $v = \frac{pzn}{60 \times 1000}$ m/sec.

Where p = pitch of the chain in mm and z = number of teeth in sprocket. This chordal action of the chain is shown in Fig. 8.17.



a. Four teeth sprocket *b. Five teeth sprocket*

Fig. 8.17

8.5.7 Design procedure for roller chain

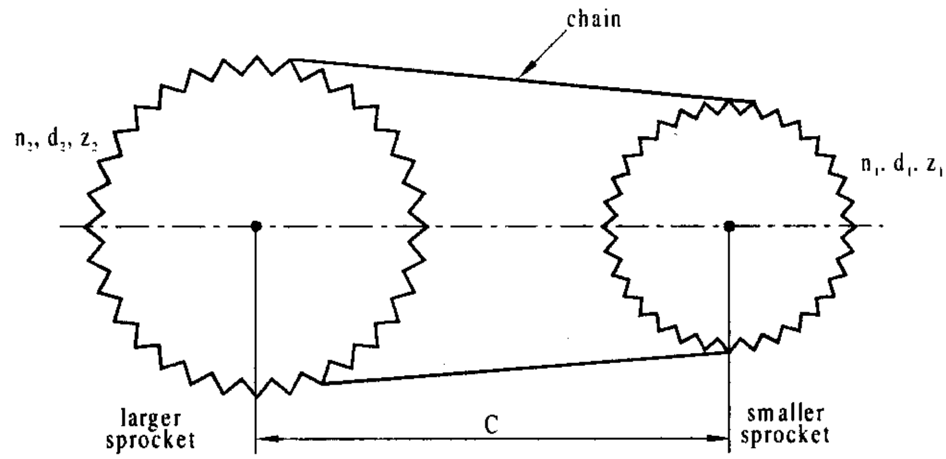


Fig. 8.18

Let p = Pitch

d_1 = Diameter of smaller sprocket

d_2 = Diameter of larger sprocket

n_1 = Speed of smaller sprocket
n_2 = Speed of larger sprocket
z_1 = Number of teeth on smaller sprocket
z_2 = Number of teeth on larger sprocket
L = Length of chain
L_p = Length of chain in pitches
C = Centre diameter
C_p = Centre distance in pitches
F_u = Ultimate or breaking load
F_θ = Required chain pull
F_a = Allowable pull
n_o = Working factor of safety
n_a = Actual factor of safety
v = Velocity

1. Pitch of chain

$$p \leq 25\left(\frac{900}{n_1}\right)^{\frac{2}{3}} \qquad \text{---- 21.106 c (DDHB)}$$

where p in mm, and n_1 = speed of smaller sprocket
Select standard nearest value of pitch from Table 21.64
Chain number
Breaking load F_u
Measuring load w

2. Number of teeth on the sprockets

From Table 21.60 for the given $\frac{n_1}{n_2}$ ratio select the number of teeth on the smaller sprocket (z_1)

$$\text{Since } \frac{n_1}{n_2} = \frac{z_2}{z_1}$$

Number of teeth on larger sprocket = z_2

3. Pitch diameters

$$d = \frac{p}{\sin\left(\frac{180}{z}\right)} \qquad \text{---- 21.126 (DDHB)}$$

$$\therefore\ d_1 = \frac{p}{\sin\left(\frac{180}{z_1}\right)} = \text{pitch diameter of smaller sprocket}$$

$$d_2 = \frac{p}{\sin\left(\frac{180}{z_2}\right)} = \text{pitch diameter of larger sprocket}$$

4. *Velocity*

$$v = \frac{pz_1n_1}{60000}\ \text{m/sec} \qquad \text{---- 21.105 (DDHB)}$$

5. *Required pull*

Power required $N = \frac{F_\theta . v}{1000k_l k_s}$ ---- 21.115a

k_l = Load factor

= 1.1 to 1.5 or from Table 14.4 (Old DDHB) or Table 14.7 (New DDHB)

k_s = Service factor

= 1 for 10 hours per day service

= 1.2 for 24 hours operation or from Table 21.61 (Old DDHB) or Table 21.54 F (New DDHB)

6. *Allowable pull*

Allowable pull $F_a = \frac{F_u}{n_o}$ ---- 21.113 (DDHB)

where n_o = working factor of safety from Table 21.75

7. *Number of strands in a chain*

Number of strands $i = \frac{F_\theta}{F_a}$ ---- 21.118 (DDHB)

8. *Check for actual factor of safety*

Actual factor of safety $n_a = \left(\frac{F_u}{F_\theta + F_{cs} + F_s}\right) i$ ---- 21.117 (DDHB)

where $F_\theta = \frac{1000N}{v}$ ---- 21.117 b (DDHB)

$F_{cs} = \frac{wv^2}{g}$ ---- 21.117 a (DDHB)

$F_s = k_{sg}\, wC$

C = centre distance. If not given for medium centre distance C_p = 30 to 50 ---- 21.119

k_{sg} = coefficient of sag from Table 21.58

i = number of strands

If $n_a > n_o$, then safe

9. Length of chain in pitches

$$L_p = 2\,C_p \cos\alpha + \frac{z_2 + z_1}{2} + \alpha\left(\frac{z_2 - z_1}{180}\right)$$

$$\text{Where } \alpha = \sin^{-1}\left(\frac{d_2 - d_1}{2C}\right) \quad \text{---- 21.122 (DDHB)}$$

10. Length of chain

$$L = p.L_p \quad \text{---- } \frac{21.125}{316} \text{ (DDHB)}$$

11. Correct centre distance

$$L_p = 2\frac{C}{p}\cos\alpha + \frac{z_2 + z_1}{2} + \alpha\left(\frac{z_2 - z_1}{180}\right)$$

$\therefore$ Correct centre distance C

Example 8.19

Select a roller chain drive to transmit power of 10 kW from a shaft rotating at 750 rpm to another shaft to run at 450 rpm. The distance between the shaft centres could be taken as 35 pitches.

BU August 1996

Data :

$$N = 10\text{ kW};\quad n_1 = 750\text{ rpm};\quad n_2 = 450\text{ rpm};\quad C = 35\text{ pitches}$$

Solution :

1. Pitch of chain

$$p \le 25\left(\frac{900}{n_1}\right)^{2/3} \quad \text{---- 21.106 c (DDHB)}$$

$$\le 25\left(\frac{900}{750}\right)^{2/3}$$

$$\le 28.23 \text{ mm}$$

From table 21.64, the nearest standard value of pitch **p = 25.4 mm**

Select chain number 208 B

Breaking load F_u = 17.9 kN = 17900 N

Measuring load w = 127.5 N

2. Number of teeth on the sprockets

$$\frac{n_1}{n_2} = \frac{750}{450} = 1.667$$

From Table 21.60 for $\frac{n_1}{n_2} = 1.667$, select number of teeth on the sumaller sprocket $z_1 = 27$

$$\text{Now } \frac{n_1}{n_2} = \frac{z_2}{z_1}$$

$$\frac{750}{450} = \frac{z_2}{27}$$

Number of teeth on larger sprocket $z_2 = 45$

3. Pitch diameter

$$d = \frac{p}{\sin\left(\frac{180}{z}\right)} \qquad \text{---- 21.126 (DDHB)}$$

Pitch diameter of smaller sprocket $d_1 = \frac{p}{\sin\left(\frac{180}{z_1}\right)} = \frac{25.4}{\sin\left(\frac{180}{27}\right)} = 218.79$ mm

Pitch diameter of larger sprocket $d_2 = \frac{p}{\sin\left(\frac{180}{z_2}\right)} = \frac{25.4}{\sin\left(\frac{180}{45}\right)} = 364.124$ mm

4. Velocity

$$v = \frac{pz_1n_1}{60000} = \frac{25.4 \times 27 \times 750}{60000} = 8.57 \text{ m/sec}$$

5. Required pull

$$\text{Power } N = \frac{F_\theta \cdot v}{1000\,k_l k_s} \qquad \text{---- 21.115a (DDHB)}$$

k_l = Load factor = 1.1 – 1.5

k_s = Service factor

= 1.2 for 24 hours operation (Assume 24 hours operation)

Take k_l = 1.3

$$\therefore 10 = \frac{F_\theta \times 8.57}{1000 \times 1.3 \times 1.2}$$

$$\therefore \mathbf{F_\theta = 1820.3\ N}$$

6. Allowable pull

$F_a = \frac{F_u}{n_o}$ where n_o = Working factor of safety

From Table 21.75 for n_t = 750 rpm and p = 25.4 mm

Select the working factor of safety $n_o = 11.7$ [n_o is not equal to 10.7, printing error in DDHB]

$$\therefore F_a = \frac{17900}{11.7} = 1529.914$$

7. Number of strands

$$i = \frac{F_\theta}{F_a} = \frac{1820.3}{1529.914} = 1.189 \qquad \text{---- 21.118 (DDHB)}$$

$\therefore$ Number of strands $i = 2$

8. Check for actual factor of safety

$$\text{Actual factor of safety } n_a = \left(\frac{F_u}{F_\theta + F_{cs} + F_s}\right) i$$

$$F_\theta = \frac{1000\,N}{v} = \frac{1000 \times 10}{8.57} = 1166.86\text{ N} \qquad \text{---- 21.117b (DDHB)}$$

$$F_{cs} = \frac{wv^2}{g} = \frac{127.5 \times 8.57^2}{9.81} = 954.56\text{ N} \qquad \text{---- 21.117a (DDHB)}$$

$$F_s = k_{sg}\, w\, C \qquad \text{---- 21.117a (DDHB)}$$

From Table 21.58 for horizontal drive, $k_{sg} = 6$

$$\therefore\ F_s = 6 \times 127.5 \times \frac{35 \times 25.4}{1000} = 680.085\text{ N}$$

$$\therefore\ n_a = \left(\frac{17900}{1166.86 + 954.56 + 680.085}\right) \times 2 = 12.778$$

Since $n_a > n_o$, the selection of the chain is safe.

9. Length of chain in pitches

$$L_p = 2\,C_p \cos\alpha + \frac{z_1 + z_2}{2} + \alpha\left(\frac{z_2 - z_1}{180}\right) \qquad \text{---- 21.122 (DDHB)}$$

$$\alpha = \sin^{-1}\left(\frac{d_2 - d_1}{2C}\right) \qquad \text{---- 21.122 (DDHB)}$$

$$= \sin^{-1}\left(\frac{364.124 - 218.79}{2 \times 35 \times 25.4}\right) = 4.6886^\circ$$

$$\therefore\ L_p = 2 \times 35 \cos 4.6886 + \left(\frac{27 + 45}{2}\right) + 4.6886\left(\frac{45 - 27}{180}\right)$$

$$= 106.2346 \text{ pitches}$$

The nearest even number of pitches is 106

$$\therefore\ \mathbf{L_p = 106 \text{ pitches}}$$

10. Length of chain

$$L = L_p \cdot p \qquad \text{---- 21.125 (DDHB)}$$

$$\therefore\ L = 106 \times 25.4 = 2692.4\text{ mm}$$

11. Correct centre distance

$$L_p = 2\frac{C}{p}\cos\alpha + \frac{(z_2 + z_1)}{2} + \alpha\frac{(z_2 - z_1)}{180}$$

$$106 = 2 \times \frac{C}{25.4}\cos 4.6886 + \left(\frac{27+45}{2}\right) + 4.6886\left(\frac{45-27}{180}\right)$$

$$\therefore C = 886 \text{ mm}$$

Example 8.20

Select a chain drive to actuate a compressor from 10 kW electric motor at 970 rpm, the compressor rpm being 350. Minimum centre distance should be approximately 560 mm. The chain tension may be adjusted by shifting the motor on rails. Compressor is to work for 10 hours/day.

BU August 1997

Data :

$$\mathbf{N = 10\ kW;\quad n_1 = 970\ rpm;\quad n_2 = 350\ rpm}$$

Approximate centre distance C = 560 mm; Working 10 hours/day.

Solution :

1. Pitch of chain

$$p \leq 25\left(\frac{900}{n_1}\right)^{2/3} \qquad \text{---- 21.106 c (DDHB)}$$

$$\leq 25\left(\frac{900}{970}\right)^{2/3} \leq 23.78 \text{ mm}$$

From Table 21.64, the nearest standard pitch **p = 25.4 mm**

Select chain number 208 B

Breaking load F_u = 17.9 kN = 17900 N

Measuring load w = 127.5 N.

2. Number of teeth on the sprockets

$$\frac{n_1}{n_2} = \frac{970}{350} = 2.77$$

From Table 21.60 for $\frac{n_1}{n_2} = 2.77$ Number of teeth on the smaller sprocket $z_1 = 26$

$$\text{Now,}\quad \frac{n_1}{n_2} = \frac{z_2}{z_1}$$

$$\frac{970}{350} = \frac{z_2}{26}$$

Number of teeth on larger sprocket $\mathbf{z_2 = 72}$

3. Pitch diameters

$$d = \frac{p}{\sin\left(\frac{180}{z}\right)} \qquad \text{---- 21.126 (DDHB)}$$

pitch diameter of smaller sprocket $d_1 = \frac{p}{\sin\left(\frac{180}{z_1}\right)} = \frac{25.4}{\sin\left(\frac{180}{26}\right)} = 210.724$ mm

pitch diameter of larger sprocket $d_2 = \frac{p}{\sin\left(\frac{180}{z_2}\right)} = \frac{25.4}{\sin\left(\frac{180}{72}\right)} = 582.31$ mm

4. Velocity

$$\text{Velocity } v = \frac{pz_1n_1}{60000} = \frac{25.4 \times 26 \times 970}{60000} = 10.6765 \text{ m/sec.}$$

5. Required pull

$$\text{Powr, } N = \frac{F_\theta . v}{1000 k_l k_s} \qquad \text{---- 21.115 a (DDHB)}$$

$$k_s = 1 \text{ for 10 hours duty/day}$$

From Table 14.4 (old DDHB) or Table 14.7 (New DDHB) for compressor $k_l = 1.75$

$$\therefore \ 10 = \frac{F_\theta \times 10.6765}{1000 \times 1.75 \times 1}$$

$$\mathbf{F_\theta = 1639.12\ N}$$

6. Allowable pull

$$F_a = \frac{F_u}{n_o}$$

From Table 21.75 for $n_1 = 970$ rpm and $p = 25.4$ mm

Select the working factor of safety $n_o = 12.9$

$$\therefore \ F_a = \frac{17900}{12.9} = 1387.6\text{N}$$

7. Number of strands

$$i = \frac{F_\theta}{F_a} = \frac{1639.12}{1387.6} = 1.18$$

$\therefore$ Number of strands $i = 2$

8. Check for actual factor of safety

$$\text{Actual factor of safety } n_a = \left(\frac{F_u}{F_\theta + F_w + F_s}\right) i \qquad \text{---- 21.117 (DDHB)}$$

$$F_0 = \frac{1000N}{v} = \frac{1000 \times 10}{10.6765} = 936.64\,N$$

$$F_{cs} = \frac{wv^2}{g} = \frac{127.5 \times 10.6765^2}{9.81} = 1481.5\,N$$

$$F_s = k_{sg}\,wC \qquad \text{---- 21.117 a (DDHB)}$$

From Table 21.58 for horizontal drive $k_{sg} = 6$

$$\therefore\ F_s = 6 \times 127.5 \times \frac{560}{1000} = 428.4\,N$$

$$\therefore\ n_a = \left(\frac{17900}{936.64 + 1481.5 + 428.4}\right) \times 2 = 12.576$$

Since $n_a \approx n_o$ the selection of the chain is safe

9. Length of chain in pitches

$$L_p = 2\,C_p \cos\alpha + \left(\frac{z_2 + z_1}{2}\right) + \alpha\left(\frac{z_2 - z_1}{180}\right) \qquad \text{---- 21.122 (DDHB)}$$

$$\alpha = \sin^{-1}\left(\frac{d_2 - d_1}{2C}\right) \qquad \text{---- 21.122 (DDHB)}$$

$$= \sin^{-1}\left(\frac{582.31 - 210.724}{2 \times 560}\right) = 19.376°$$

$$\therefore\ L_p = 2 \times \frac{560}{25.4}\cos 19.376 + \left(\frac{26 + 72}{2}\right) + 19.376\left(\frac{72 - 26}{180}\right) = 95.548$$

$\therefore$ The nearest even number of pitches = 96

$$\therefore\ \mathbf{L_p = 96\ pitches}$$

10. Length of chain

$$L = p.L_p \qquad \text{---- 21.125 (DDHB)}$$

$$= 25.4 \times 96 = 2438.4\,N$$

11. Correct centre distance

$$L_p = 2 \times \frac{C}{p}\cos\alpha + \left(\frac{z_2 + z_1}{2}\right) + \alpha\left(\frac{z_2 - z_1}{180}\right)$$

$$96 = 2 \times \frac{C}{25.4}\cos 19.376 + \left(\frac{26 + 72}{2}\right) + 19.376\left(\frac{72 - 26}{180}\right)$$

$$\mathbf{C = 566\ mm}$$

Example 8.21

Select a chain drive to actuate a compressor from 15 kW electric motor at 600 rpm, the compressor rpm being 120.

Data :

$$\mathbf{N = 15\,kW;\ n_1 = 600\,rpm;\ n_2 = 120\,rpm}$$

Solution :

1. Pitch of chain

$$p \leq 25\left(\frac{900}{n_1}\right)^{2/3} \qquad \text{---- 21.106 c (DDHB)}$$

$$\leq 25\left(\frac{900}{600}\right)^{2/3}$$

$$\leq 32.75 \text{ mm}$$

From Table 21.64, the nearest standard value of pitch **p = 31.70 mm** Select chain Number 210B
Breaking load F_u = 22.3 kN = 22300 N; Measuring load w = 196.1 N

2. Number of teeth on the sprockets

$$\frac{n_1}{n_2} = \frac{600}{120} = 5$$

From table 21.60 for i = 5

Number of teeth on the smaller sprocket $z_1 = 21$

$$\therefore \frac{n_1}{n_2} = \frac{z_2}{z_1}$$

$$\frac{600}{120} = \frac{z_2}{21}$$

∴ Number of teeth on the larger sprocket $\mathbf{z_2 = 105}$

3. Pitch diameters

$$d = \frac{p}{\sin\left(\frac{180}{z}\right)} \qquad \text{---- 21.126 (DDHB)}$$

∴ Pitch diameter of smaller sprocket $d_1 = \dfrac{p}{\sin\left(\frac{180}{z_1}\right)} = \dfrac{31.70}{\sin\left(\frac{180}{21}\right)} = 212.69$ mm

Pitch diameter of larger sprocket $d_2 = \dfrac{p}{\sin\left(\frac{180}{z_2}\right)} = \dfrac{31.70}{\sin\left(\frac{180}{105}\right)} = 1059.65$

4. Velocity

$$v = \frac{pz_1n_1}{60000} = \frac{31.70 \times 21 \times 600}{60000} = 6.657 \text{ m/sec}$$

5. Required pull

$$N = \frac{F_0 \cdot v}{1000k_l k_s} \qquad \text{---- 21.115 a (DDHB)}$$

Assume 24 hours duty/day $\therefore$ $k_s = 1.2$

From Table 14.4 (old DDHB) or Table 14.7 (New DDHB) for compressor $k_l = 1.75$

$$\therefore\ 15 = \frac{F_0 \times 6.657}{1000 \times 1.75 \times 1.2}$$

$$\therefore\ \mathbf{F_0 = 4731.8612\ N}$$

6. Allowable pull

From Table 21.75 for $n_1 = 600$ rpm and $p = 31.70$ mm $n_o = 13.2$

$$F_a = \frac{F_u}{n_o} = \frac{22300}{13.2} = 1689.4\ \text{N} \quad \text{---- 21.113 (DDHB)}$$

7. Number of strands

$$i = \frac{F_0}{F_a} = \frac{4731.8612}{1689.4} = 2.8 \quad \text{---- 21.118 (DDHB)}$$

$\therefore$ Number of strands $i = 3$

8. Check for actual factor of safety

$$\text{Actual factor of safety } n_a = \left(\frac{F_u}{F_0 + F_{Cs} + F_s}\right) i \quad \text{---- 21.117 (DDHB)}$$

$$F_0 = \frac{1000N}{v} = \frac{1000 \times 15}{6.657} = 2253.27\ \text{N}$$

$$F_{Cs} = \frac{wv^2}{g} = \frac{196.1 \times 6.657^2}{9.81} = 885.861\ \text{N}$$

$$F_s = k_{sg}\, w\, C \quad \text{---- 21.117 a (DDHB)}$$

From Table 21.58 for horizontal drive $k_{sg} = 6$

For medium centre distance $C = 30\,p$ to $50\,p$ ---- 21.119 (DDHB)

$$C_{min} = 30\,p = 30 \times 31.70 = 957\ \text{mm}$$

$$C_{max} = 50\,p = 50 \times 31.70 = 1585\ \text{mm}$$

$$\text{Take } C = 1000\ \text{mm}$$

$$\therefore\ F_s = 6 \times 196.1 \times \frac{1000}{1000} = 1176.6\ \text{N}$$

$$\therefore\ n_u = \left(\frac{22300}{2253.27 + 885.861 + 1176.6}\right) \times 3 = 15.5$$

Since $n_a > n_o$, the selection of the chain is safe.

9. Length of chain in pitches

$$L_p = 2\,C_p \cos\alpha + \left(\frac{z_2 + z_1}{2}\right) + \alpha\left(\frac{z_2 - z_1}{180}\right) \quad \text{---- 21.122 (DDHB)}$$

$$\alpha = \sin^{-1}\left(\frac{d_2 - d_1}{2C}\right) = \sin^{-1}\left(\frac{1059.65 - 212.69}{2 \times 1000}\right)$$

$$\alpha = 25.054^\circ$$

$$\therefore\ L_p = 2\times\frac{1000}{31.70}\cos 25.054+\left(\frac{21+105}{2}\right)+25.054\left(\frac{105-21}{180}\right)=131.847$$

The nearest even number of pitches is 132

$$\therefore\ L_p = 132 \text{ pitches}$$

10. Length of chain

$$L = p.L_p = 31.70\times 132 = 4184.4 \text{ mm} \quad \text{---- 21.125 (DDHB)}$$

11. Correct centre distance

$$L_p = 2\times\frac{C}{p}\cos\alpha+\left(\frac{z_2+z_1}{2}\right)+\alpha\left(\frac{z_2-z_1}{180}\right)$$

$$132 = 2\times\frac{C}{31.7}\cos 25.054+\left(\frac{21+105}{2}\right)+25.054\left(\frac{105-21}{180}\right)$$

$$\therefore\ C = 1002.6767 \text{ mm}$$

Example 8.22

A roller chain is to transmit 66.24 kW from a 17 tooth sprocket to a 34 tooth sprocket at a pinion speed of 300 rpm. The loads are moderate shock. The equipment is to run 18 hours/day. Specify the length and size of the chain required for a center distance of about 25 pitches VTU, March 2001

Data :

$N = 66.24$ kW; $z_1 = 17$; $z_2 = 34$; $n_1 = 300$ rpm;
Moderate shock and runs 18 hours/day; $C_p = 25$

Solution :

1. Pitch of chain

$$p \le 25\left(\frac{900}{n_1}\right)^{2/3} \quad \text{---- 21.106 c (DDHB)}$$

$$\le 25\left(\frac{900}{300}\right)^{2/3}$$

$$\le 52 \text{ mm}$$

From Table 21.64 the nearest standard value of pitch $p = 50.8$ mm

Select chain number : 216 A

Breaking load $F_u = 55.6$ kN $= 55600$ N

Measuring load $w = 500.2$ N

2. Speed of larger sprocket

$$\frac{n_1}{n_2} = \frac{z_2}{z_1}$$

$$\frac{300}{n_2} = \frac{34}{17} \quad \therefore\ n_2 = 150 \text{ rpm}$$

3. Pitch diameter

$$d = \frac{p}{\sin\left(\frac{180}{z}\right)} \qquad \text{---- 21.126 (DDHB)}$$

Pitch diameter of smaller sprocket $d_1 = \frac{p}{\sin\left(\frac{180}{z_1}\right)} = \frac{50.8}{\sin\left(\frac{180}{17}\right)} = 276.46$ mm

Pitch diameter of larger sprocket $d_2 = \frac{p}{\sin\left(\frac{180}{z_2}\right)} = \frac{50.8}{\sin\left(\frac{180}{34}\right)} = 550.57$ mm

4. Velocity

$$v = \frac{pz_1n_1}{60000} = \frac{50.8 \times 17 \times 300}{60000} = 4.318 \text{ m/sec}$$

5. Required pull

$$\text{Power } N = \frac{F_0 . v}{1000k_l k_s} \text{ kW} \qquad \text{---- 21.115 a (DDHB)}$$

k_l = load factor = 1.1 – 1.5 ; Take $k_l = 1.3$

From Table 21.61 (Old DDHB) or Table 21.54F (New DDHB) for moderate shock, 18 hours/day

$$k_s = 1.5 - 2.4, \text{take } k_s = 2$$

$$\therefore\ 66.24 = \frac{F_0 \times 4.318}{1000 \times 1.3 \times 2} \quad \therefore\ F_0 = 39885.132 \text{ N}$$

6. Allowable pull

$$F_a = \frac{F_u}{n_o} \qquad \text{---- 21.113 (DDHB)}$$

From Table 21.75 for n_1 = 300 rpm and p = 50.8 mm

Working factor of safely n_o = 13.2

$$\therefore\ F_a = \frac{55600}{13.2} = 4212.12 \text{ N}$$

7. Number of strands

$$i = \frac{F_0}{F_a} = \frac{39885.132}{4212.12} = 9.496$$

$\therefore$ Number of strands i = 10

8. Check for actual factor of safety

$$n_a = \left(\frac{F_u}{F_\theta + F_{Cs} + F_s}\right) i \qquad \text{---- 21.117 (DDHB)}$$

$$F_\theta = \frac{1000 \times N}{v} = \frac{1000 \times 66.24}{4.318} = 15340.435 \text{ N}$$

$$F_{Cs} = \frac{wv^2}{g} = \frac{500.2 \times 4.318^2}{9.81} = 950.692 \text{ N}$$

$$F_s = k_{sg}\, w\, C$$

From Table 21.58 for horizontal drive $k_{sg} = 6$

$$\therefore \quad F_s = 6 \times 500.2 \times \left(\frac{25 \times 50.8}{1000}\right) = 3811.524 \text{ N}$$

$$\therefore \quad n_a = \left(\frac{55600}{15340.435 + 950.692 + 3811.524}\right)10 = 27.65$$

Since $n_a > n_o$, design is safe

9. Length of chain in pitches

$$L_p = 2\,C_p \cos\alpha + \frac{z_2 + z_1}{2} + \alpha\left(\frac{z_2 - z_1}{180}\right) \qquad \text{---- 21.122 (DDHB)}$$

$$\alpha = \sin^{-1}\left(\frac{d_2 - d_1}{2C}\right) = \sin^{-1}\left(\frac{550.57 - 276.46}{2 \times 25 \times 50.8}\right) = 6.195°$$

$$\therefore \quad L_p = 2 \times 25 \cos 6.195 + \left(\frac{34 + 17}{2}\right) + 6.195\left(\frac{34 - 17}{180}\right) = 75.7$$

The nearest even number of pitches is 76

$$\therefore \quad L_p = 76 \text{ pitches.}$$

10. Length of chain

$$L = L_p.p = 76 \times 50.8 = 3860.8 \text{ mm}$$

11. Correct centre distance

$$L_p = 2\frac{C}{p}\cos\alpha + \left(\frac{z_2 + z_1}{2}\right) + \alpha\left(\frac{z_2 - z_1}{180}\right)$$

$$76 = 2 \times \frac{C}{50.8}\cos 6.195 + \left(\frac{34 + 17}{2}\right) + 6.195\left(\frac{34 - 17}{180}\right)$$

$$C = 1275.286 \text{ mm}$$

Silent Chain

Example 8.23

A ventilating fan operating at 400 rpm requires 7.5 kW under rated conditions. The fan is to be installed in a conference hall where quietness of operation is essential. The fan is to be connected to an induction motor by means of chain drive. Use the following data : Motor speed = 1200 rpm; Number of teeth on the smaller sprocket = 20; efficiency of the drive = 90%. Determine the following
(i) Power of the motor required; (ii) Type of chain; (iii) Number of teeth in the larger sprocket;
(iv) The pitch and width of chain; (v) Length of chain.

Data :

$$n_1 = 1200 \text{ rpm};\; n_2 = 400 \text{ rpm};\; \eta = 90\%;\; z_1 = 20$$

Solution :

i. Power of motor

$$\text{Power of motor } N = \frac{\text{Required power}}{\text{Efficiency}} = \frac{7.5}{0.9} = 8.33 \text{ kW}$$

ii. Type of chain

Since quietness of operation is essential silent chain is recommended.

iii. Number of teeth on the larger sprocket

$$\frac{n_1}{n_2} = \frac{z_2}{z_1}$$

$$\frac{1200}{400} = \frac{z_2}{20} \quad \therefore \text{ Number of teeth on the larger sprocket } z_2 = 60$$

iv. Pitch and width of chain

$$\text{Pitch } p \le 25\left(\frac{900}{n_1}\right)^{2/3} \qquad \text{---- 21.106 c (DDHB)}$$

$$\le 25\left(\frac{900}{1200}\right)^{2/3}$$

$$\le 20.63 \text{ mm}$$

From Table 21.73

$$\text{Standard pitch } p = 19 \text{ mm}$$

$$\text{Centre distance } C = 381 \text{ mm}$$

$$\text{Velocity } v = \frac{pz_1n_1}{60000} = \frac{19\times 20\times 1200}{60000} = 7.6 \text{ m/sec} \qquad \text{---- 21.105 (DDHB)}$$

$$\text{Power transmitted per mm of width } N = \frac{pv}{680}\left[1-\frac{v}{2.16(z_1-8)}\right]\times 0.736 \text{ kW} \qquad \text{---- 21.132 (DDHB)}$$

$$= \frac{19\times 7.6}{680}\left[1-\frac{7.6}{2.16(20-8)}\right]\times 0.736 = 0.1104655 \text{ kW}$$

$$\therefore \text{ Width of chain } w = \frac{\text{Total power}}{\text{Power / mm of width}} = \frac{8.33}{0.1104655} = 75.408 \text{ mm}$$

$$= 75.5 \text{ mm}$$

v. Length of chain

$$d = \frac{p}{\sin\left(\frac{180}{z}\right)} \qquad \text{---- 21.126 (DDHB)}$$

$$\therefore\ d_1 = \frac{p}{\sin\left(\frac{180}{z_1}\right)} = \frac{19}{\sin\left(\frac{180}{20}\right)} = 121.46\,\text{mm}$$

$$d_2 = \frac{p}{\sin\left(\frac{180}{z_2}\right)} = \frac{19}{\sin\left(\frac{180}{60}\right)} = 363.04\ \text{mm}$$

$$\alpha = \sin^{-1}\left(\frac{d_2 - d_1}{2C}\right) = \sin^{-1}\left(\frac{363.04 - 121.46}{2\times 381}\right) = 18.484°$$

$$L = 2C\cos\alpha + \frac{z_1 p(180+2\alpha)}{360} + \frac{z_2 p(180-2\alpha)}{360} \quad \text{---- 21.124 (DDHB)}$$

$$= 2 \times 381 \cos 18.484 + \frac{20\times 19(180+2\times 18.484)}{360} + \frac{60\times 19(180-2\times 18.484)}{360} = 1404.665\ \text{mm}$$

EXERCISES

1. Select a V-belt drive to transmit a power of 8 kW from a shaft rotating at 1000 rpm to a parallel shaft to be run at 400 rpm. Space restrict the pitch diameter of the smaller pulley to 150.0 mm
2. A weight of 12 kN is to be raised to a height of 500 m with velocity of 10 m/mt. This velocity is to be achieved while the weight traverses through a distance of 20 m. Select a suitable size of a wire rope required for the purpose.
3. Write a note on Rockwood drive.
4. Select a flat-belt drive for a compressor running at 720 rpm, which is driven by a 25 kW, 1440 rpm motor. Space is available for a centre distance of 3m. The belt is open type.
5. A compressor requiring 90 kW, is to run at about 250 rpm. The drive is achieved by V belts from an electric motor running 750 rpm. The diameter of the pulley on the compressor shaft must not be greater than 1m. While the centre distance between the pulleys is limited to 1.75 m. The belt speed should not exceed 1600 m/minute. Determine the number of V belts required to transmit the power if each belt has a cross sectional area of 375 mm^2. The density of the belt material is 1000 kg/m^3 and has an allowable tensile stress of 2.5 N/mm^2. The groove of the pulley is has an angle of 35°. The coefficient of friction between the belt and pulley is 0.25. Calculate the length of belt required.
6. Select a suitable wire rope to lift a load of 10 kN of debris from a well 60 m deep. The rope should have a factor of safety of 6. The weight of the bucket is 5 kN. The load is lifted up with minimum speed of 150 m/min. Which is obtained in one second. Find also the stress

induced in the rope due to starting with an initial slack of 250 mm. The average tensile strength of the rope may be taken as $590d^2$ Newtons (where d is the rope diameter in mm for a 6×19 wire rope. The weight of the rope is 18.5 N/m. Take the diameter of the wire $d_w = 0.063$ d and area of the rope $A = 0.38d^2$.

7. Following are the details of flat belt drive connecting a motor and Fan.

	Motor	Fan
Pulley Diameter	400 mm	1600 mm
Angle of Wrap	3.78 rad	3.78 rad
Coefficient of friction	0.30	0.25
Speed	700 rpm	175 rpm

The belt is 5 mm thick and has a density of 10 kN/m³. Allowable tensile stress in the belt is 2.0 MPa. Calculate the width of the belt.

8. Calculate the rope diameter for a 6×19 wire rope to lift a load of 10 kN of debris from a well of 60.25 m deep. The weight of the bucket is 5 kN and it is lifted at a maximum speed of 150 m/min, which is attained in 1.0 second. There is no slack in the rope. The sheave diameter is 60 times the rope diameter. The average tensile strength of the rope is $590d^2$N, 'd' is the diameter of the rope. The factor of safety 5.0. The corrected Young's modulus for the rope is 82.8×10^3 MPa.

9. a) Select a V belt drive to transmit a power of 6 kW from a shaft rotating at 1500 rpm to a parallel shaft to be run at 375 rpm. The distance between the shaft centers is 500 mm. The pitch diameter of the smaller grooved pulley can be taken to be 150 mm. The factor of application is to be taken as 1.2.

   b) Select a wire rope to lift a load of 10 kN by 200 m. The desired velocity of 20 m/ min is to be achieved while traveling through a distance of 10 m. **(VTU, Jan/Feb. 2005)**

10. a) Select a V belt drive to transmit a load of 6 kW from a shaft rotating at 1000 rpm to a parallel shaft to be rotated at 350 rpm. The space limits the center distance between shafts to 500 mm. The pitch diameter of the smaller pulley could be assumed to be 150mm.

    b) Select a suitable wire rope of a standard strand to raise a load of 10 kN through 400m. The load has to achieve a desired linear speed of 20 m/min while traversing through a distance of 15m from the start. **(VTU, Jan/Feb. 2006)**

11. a. Show that in flat belt drives the ratio of belt tensions is given by = $\frac{T_1}{T_2} = e^{\mu\theta}$, where T_1 and T_2 are belt tensions, μ is the coefficient of friction and θ is the angle of wrap.

    b. Select the type and number of V-belts required to drive a crusher, which works 8 hours a day. The power transmitted is 65 kW. The motor shaft runs at 900 rpm and carries a pulley of 250mm diameter, the crusher shaft rotates at 300 rpm and the centre distance is 700 mm. Determine the pitch length of the belt required. **(VTU, July 2006)**

12. a) Select a V belt drive to transmit 10 kW of power from a pulley of 200mm diameter mounted on an electric motor running at 720 rpm to another pulley mounted on compressor running at 200 rpm. The approximate centre distance between the two pulleys is 600mm. The correction factor for service is 1.3. Find the number of belts and the correct centre distance.

    b) Select a suitable wire rope of a standard strand to lift a load of 10kN through a height of 600m from a mine. The weight of bucket is 2.5kN. The load should attain a maximum speed of 50m/min in 2 seconds. **(VTU, Dec. 06/Jan. 2007)**

13. a) Derive the equation $\frac{T_1}{T_2} = e^{\mu\theta}$, where T_1 = Tension in the belt on the tight side, T_2 = Tension in the belt on the slack side, μ = The coefficient of friction between the belt and pulley, θ = Angle of contact in radians.

    b) A compressor, requiring 85 kW, is to run at 250 rpm. The drive is by V-belts from an electric motor running at 800 rpm. The diameter of the pulley on the compressor shaft must not be grater than 1.0 meter while center distance between the pulleys is limited to 1.8 meters. The belt speed should not exceed 1500 m/min. Determine the number of V-belts required to transmit the power if each belt has a cross-sectional area of 360 mm^2, density 1000 kg/m^3 and an allowable tensile stress of 2.5 N/mm^2. The groove angle of pulley is 35°. The coefficient of friction between the belt and the pulley is 0.25. **(VTU, July 2007)**

14. a) Derive an expression for the ratio of tension in V-belt drive.

    b) Two shafts one meter apart are connected by a V-belt drive to transmit 90 kW at 1200rpm of a driver pulley of 300 mm effective diameter. The driver pulley rotates at 400rpm. The angle of groove is 40° and coefficient of friction between the belt and the pulley rim is 0.25. Area of the belt section is 400 mm^2 and the permissible stress is 2.1 MPa. Density of belt material = 1100kg/m^3. Calculate the number of belts required and the length of the belt. **(VTU, Dec. 07/Jan. 2008)**

15. A 25mm diameter 6 x 37 steel rope is used in a mine of 80 meters deep. The velocity of the cage is 2 m/sec and the time required to accelerate the cage to the desired velocity is 10 seconds. The diameter of the drum is 1.25 m. Determine the safe load that the hoist can handle by assuming a factor of safety as 8. **(VTU, June/July 2008)**

16. a) A flat belt drive is required to transmit 20 kW of power at 1440 rpm. The driver and driven shafts are approximately 3.0 m apart. The dimensions of pulley mounted on driver and driven shafts are 300 and 450 respectively. The weight density of selected belt material is 9.7 x 104 N/m3, allowable design stress σ_d= 2 N/mm2 and coefficient of friction = 0.3.

    Determine:

    i) Width of the belt if thicknes = 6mm　　ii) Length of the belt

    iii) Initial tension in the belt　　iv) Centrifugal tension in the belt.

b) Select type of V-belt and number of belts required for 10kW, 750 rpm induction motor to drive an exhaust fan in a steel plant at 250 rpm. The minimum centre distance between shafts is 1.2m. Pitch diameter of motor pulley is 200 mm.

(VTU, Dec. 08/Jan. 2009)

B.E. Degree Examination, January/February 2005
ME/AU/IP
Design of Machine Elements - II

Time : 3 Hours **Max. Marks : 100**

Note : 1. Answer any FIVE full questions.
2. Assume data suitably, if necessary.
3. Use of machine design data hand book is permitted.

1. Determine of value of t in the cross section of a curved beam shown in Fig.1 such that the normal stresses due to bending at the extreme fibers are numerically equal.

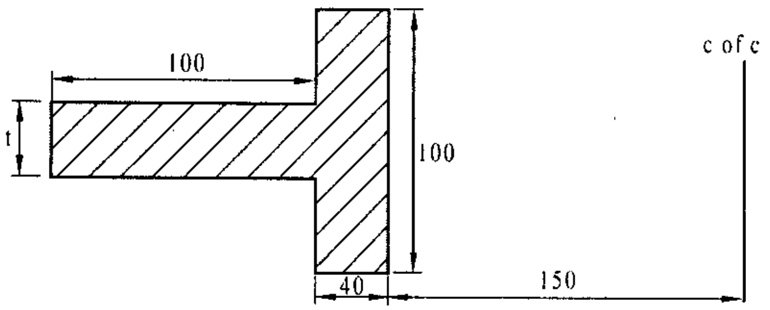


*Fig.*1

2. a) Design a helical compression spring to sustain an axial load that fluctuates between 1.5 kN and 2 kN with an associated deflection of 15 mm during the fluctuation of load. (10 Marks)

   b) An automotive leaf spring is to be designed to consist of 10 graduated leaves and 2 full length leaves. The spring is to support a central load of 5 kN over a span of 1100 mm with the central band width of 100 mm. The ratio of total depth of spring to its width is to be 2.5. Determine the width and thickness of leaves limiting the maximum equalized stress induced in the leaves to 350 MPa. Also determine the initial gap to be provided between the full length and graduated leaves before the assembly. (10 Marks)

3. a) Select a V belt drive to transmit a power of 6 kW from a shaft rotating at 1500 rpm to a parallel shaft to be run at 375 rpm. The distance between the shaft centers is 500 mm. The pitch diameter of the smaller grooved pulley can be taken to be 150 mm. The factor of application is to be taken as 1.2. (10 Marks)

   b) Select a wire rope to lift a load of 10 kN by 200 m. The desired velocity of 20 m/ min is to be achieved while traveling through a distance of 10 m. (10 Marks)

4. a) Design a cone clutch to transmit a power of 40 kW at a rated speed of 750 rpm. Also determine the

i) The axial force capacity

ii) The axial force necessary to transmit the torque

iii) The axial force necessary to engage the cone clutch. (10 Marks)

b) A simple band brake shown in Fig.2 is to be designed to absorb a power of 30 kW at a rated speed of 750 rpm. Determine :

i) The effort required to stop clockwise rotation of the brake drum.

ii) The effort required to stop counter clockwise rotation of the brake drum.

iii) The dimensions of the rectangular cross-section of the brake lever assuming its depth to be twice the width.

iv) The dimension of the cross-section of the band assuming its width to be ten times the thickness. (10 Marks)

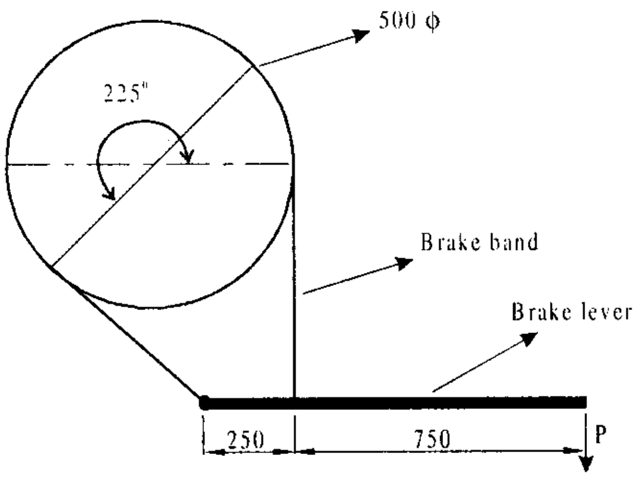


Fig.2

5. Design a pair of spur gears to transmit a power of 18 kW from a shaft running at 1000 rpm to a parallel shaft to be run at 250 rpm maintaining a distance of 160 mm between the shaft centers. Suggest suitable surface hardness for the gear pair. (20 Marks)

6. Design a pair of helical gears to transmit a power of 20 kW from a shaft running at 1500 rpm to a parallel shaft to be run at 450 rpm. Suggest suitable surface hardness for the gear pair. (20 Marks)

7. Design a pair of right-angled bevel gears to transmit a power of 15 kW from a shaft running at a speed of 7500 rpm to a perpendicular to be run at shaft 250 rpm. Suggest suitable surface hardness for the gear pair. (20 Marks)

8. a) Explain with sketches theory of hydrodynamic lubrication. (6 Marks)

b) A journal bearing is required to be designed for a rotary compressor for operation at a speed of 1500 rpm. The bearing is to sustain a load of 4500 N and the diameter of the main shaft is 50 mm. Determine

i) Length and inner diameter of the bearing bush
ii) Viscosity of an oil to be used as a lubricant and hence suggest a lubricating oil
iii) The coefficient of friction
iv) Heat generated
v) Heat dissipating capacity
vi) Amount of heat to be removed by artificial cooling
vii) Sommerfeld number. (14 Marks)

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B.E. Degree Examination, July/August 2005
ME/AU/IP
Design of Machine Elements - II

Time : 3 Hours **Max. Marks : 100**

Note : 1. Answer any FIVE full questions.
2. Assume data suitably, if necessary.
3. Use of machine design data hand book is permitted.

1. The section of a crane hook is trapezium, whose inner and outer sides are 80 mm and 40 mm respectively and has a depth of 100. The center of curvature of the section is at a distance of 120 mm from the inner side of the section and load line is 110 mm from the same point. Find the maximum load the hook can carry, if the maximum stress is not exceed 70 MPa.

2. a) Following particulars refer to a valve spring, assume static loading. Ends are ground and flat.

   Length when valve is open = 41 mm

   Length when valve is closed = 49 mm

   Spring load when valve is open = 360 N

   Spring load when valve is closed = 220 N

   Maximum inside diameter of spring is not to exceed 25 mm.

   Take $G = 8.3 \times 10^4\ MPa$. Find diameter, number of turns and pitch of the coil. The allowable shear stress is 650 MPa. (15 Marks)

   b) Derive the expression for the stress in graduated leaf spring of a Semi-elliptic springs without pre stress. (5 Marks)

3. a) A differential band brake shown in figure.1 is absorbing torque from an engine assembly. The width and thickness of band are 100 mm and 3 mm respectively. Permissible tension in the band is limited to 50 MPa. The engine is rotating at 900 rpm. Calculate following

   i) Tension in the band

   ii) The actuating force

   iii) Torque capacity of brake. (10 Marks)

   b) A multi disc plate clutch consists of 5 steel plates and 4 bronze plates. The inner and outer diameters of the friction disc are 75 mm and 150 mm respectively. The coefficient of friction is 0.1. Intensity of pressure is limited to 0.3 MPa. Assuming uniform wear theory calculate the following :

   i) The required operating force

   ii) The power transmitting capacity at 750 rpm. (10 Marks)

4. Design a pair of spur gears to transmit 27 kW for an oil pump with the gear ratio of 3.1. The rpm of the pinion is 1200. The center distance is 400 mm. The gears are to be of forged and untreated with $14\frac{1}{2}$°FDI. Check the design for dynamic and wear considerations. (20 Marks)

5. a) Explain formative number of teeth in helical gears. (5 Marks)

   b) Design a pair of helical gears to transmit 73.5 kW power with a velocity of 4.25:1. The pinion rotates at 1750 rpm. Helix angle is 15°, the teeth are 20° FDI form. The gears are lubricated randomly. Check for beam strength of tooth only. Check for beam strength of tooth only. Calculate gear forces. (15 Marks)

6. Design a pair of straight bevel gears to transmit 2 kW power at 1200 rpm. Tooth form is $14\frac{1}{2}$°FDI. The velocity ratio is 4:1. The pinion is made of forged steel heat-treated and gear material is of CI grade 35. (20 Marks)

7. a) Derive Petroff's equation for a lightly loaded bearing. (5 Marks)

   b) A full journal bearing 90 mm diameter and 150 mm long had a radial load of 2 MPa per unit projected area. Shaft speed is 500 rpm. The bearing is operating with SAE20 oil at 50°C. The specific gravity of the oil at the operating temperature is 0.985. Calculate the following :

   i) The minimum film thickness

   ii) Heat due to friction

   iii) Whether artificial cooing is necessary? (15 Marks)

8. a) Select a standard V-belt to transmit a power of 30 kW from an AC induction motor rotating at 1500 rpm to a centrifugal pump rotating at 750 rpm. The drive operates continuously for 8 hours per day. Calculate the number of belts. (10 Marks)

   b) Select a 6 × 19 steel rope to life 15 kN of debris from a tunnel 200 m deep. The bucket weighs 8 kN. The velocity of the rope is 100 m/min to be attained in 20 seconds. What will be the maximum load on the rope when there is a slack of 10 m in the rope? (10 Marks)

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B.E. Degree Examination, January/February 2006
Automobile Engineering
Design of Machine Elements - II

Time : 3 Hours **Max. Marks : 100**

Note : *1. Answer any FIVE full questions.*
2. Use of design data hand book is permitted.
3. Missing data may be suitably assumed.

1. Determine a safe value for load P for a machine element loaded as shown in figure 1, limiting the maximum normal stress induced on the cross section XX to 120 MPa.

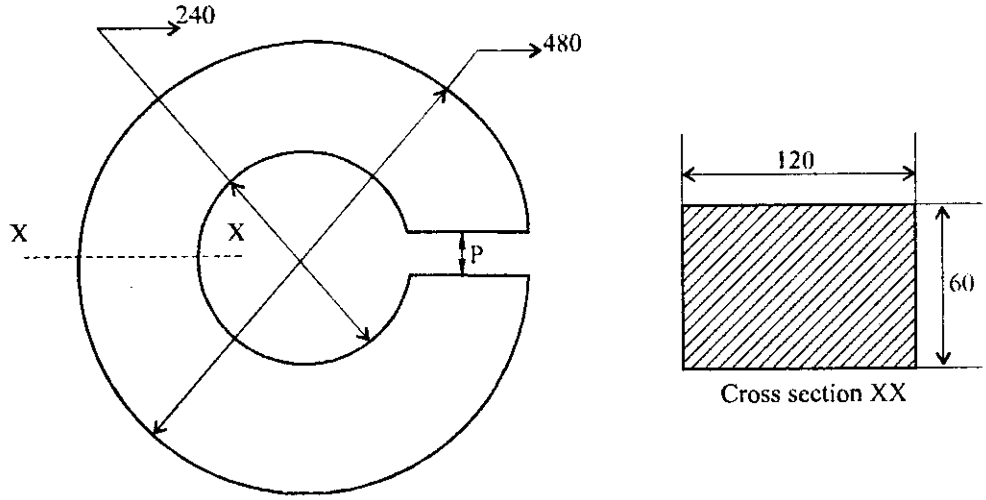

***Fig.*1**

2. a) Design a helical compression spring required for a spring loaded safety value mounted on a pressure vessel. The spring is subjected to an initial compression of 50 mm at the time of assembly and will open by 10 mm when the pressure approaches 6 MPa. The diameter of the valve is 25 mm. (10 Marks)

   b) A semi elliptical laminated leaf spring with two full length leaves and ten graduated leaves are to be designed to support a central load of 6kN over two points 1000 mm apart. The central band width is 100 mm. The ratio of total depth of the spring to its width is 2.5. The design normal stress of the material of the leaves is 400 MPa and the modulus of elasticity is 208 GPa. Determine:

   i) Width and thickness of leaves

   ii) The initial gap between full length and graduated leaves

   iii) The central bolt load. (10 Marks)

3. a) Select a V belt drive to transmit a load of 6 kW from a shaft rotating at 1000 rpm to a parallel shaft to be rotated at 350 rpm. The space limits the center distance between shafts to 500 mm. The pitch diameter of the smaller pulley could be assumed to be 150mm. (10 Marks)

   b) Select a suitable wire rope of a standard strand to raise a load of 10 kN through 400m. The load has to achieve a desired liner speed of 20 m/min while traversing through a distance of 15m from the start. (10 Marks)

4. a) Design a cone clutch to transmit a power of 40 kW at a rated speed of 750 rpm. Also determine

   i) Axial force necessary to transmit torque

   ii) Axial force necessary to engage the cone clutch. (10 Marks)

   b) A simple band brake shown in figure.2 is to be designed to stop the rotation of a shaft transmitting a power of 45 kW at rated speed of 500 rpm. Selecting suitable materials determine

   i) dimensions of the rectangular cross section of the band

   ii) dimensions of the rectangular cross section of the brake lever.

   iii) diameter of the fulcrum pin. (10 Marks)

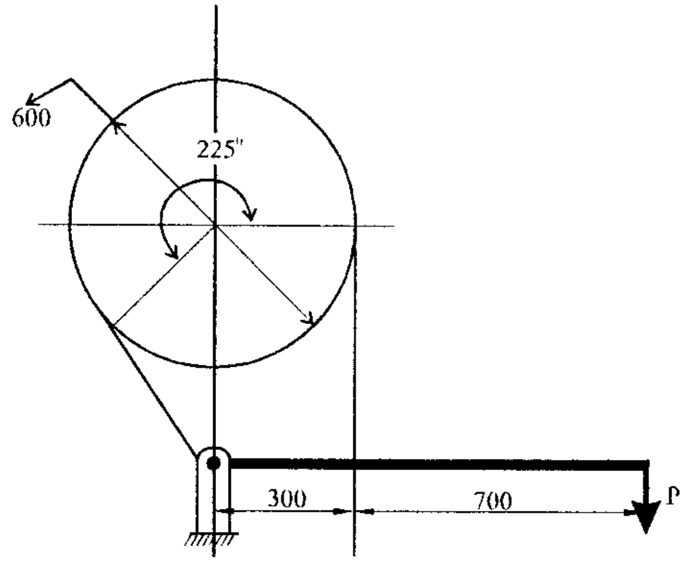

Fig.2

5. A shaft rotating at a speed of 1000 rpm is to transmit a power of 40 kW to a parallel shaft to be rotated at 350 rpm. The distance between the shaft centers is 160 mm. Design a pair of spur gears to connect these two shafts. (20 Marks)

6. Design a pair of helical gears to transmit a power of 30 kW from a shaft rotating at 1500 rpm to a parallel shaft to be rotated at 450 rpm. (20 Marks)

7. Design a pair of bevel gears to transmit a power of 25 kW from a shaft rotating at 1200 rpm to a perpendicular shaft to be rotated at 400 rpm. (20 Marks)

8. a) Derive Petroff's equation for coefficient of friction of a lightly loaded journal bearing. (6 Marks)

   b) It is required to design a main bearing of a four stroke oil engine to sustain a load of 6 kN over a shaft of diameter 50mm. The operating speed of the shaft is 1000 rpm and the operating temperature 50° C. Determine

   i) Dimensions of the bearing.

   ii) The viscosity of the oil to be used for the bearing and hence suggest appropriate oil.

   iii) Coefficient of friction

   iv) Heat generated

   v) Heat dissipated

   vi) Heat to be removed by the artificial cooling if necessary

   vii) Sommerfeld number. (14 Marks)

B.E. Degree Examination, July 2006

ME/IP/IM/MA/AU

Design of Machine Elements - II

Time : 3 Hours] **[Max. Marks : 100**

Note : 1. Answer any FIVE full questions.
2. Use of Design Data Handbook is permitted.
3. Missing data may be suitably assumed.

1. a. Explain why curved beams have to be analysed for stresses specially when we already have straight beam equations for determining the stresses? (04 Marks)

   b. The beam shown in figure Q1 (b) is subjected to a load of 50 kN. Determine the stresses at the inner and outer fibers. Plot the stress distribution. (16 Marks)

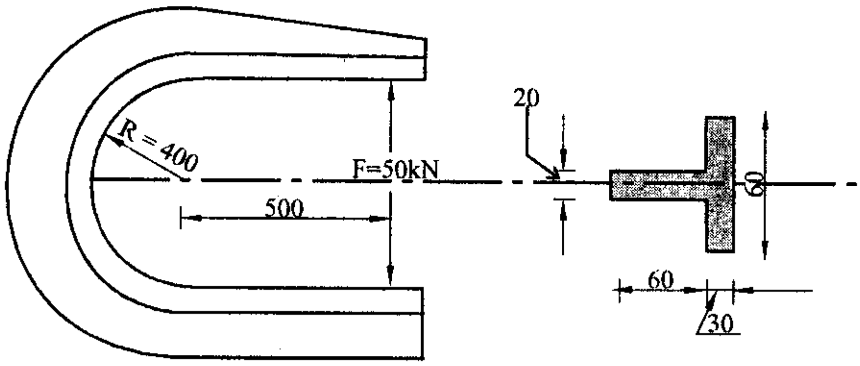

Fig. Q1 (b)

2. a. Explain about stress concentration in helical coil springs, how is it being taken care of? (03 Marks)

   b. What is surging in springs and how it can be overcome? (03 Marks)

   c. The spring used in an automobile engine has to exert 500N when the value is closed and 600N when the valve is open. The displacement of the valve is 5mm. The engine crankshaft rotates at 8000 rpm. Design the spring if permissible stress in the material of the spring is 300MPa. The ratio of mean coil diameter to the wire diameter is 6. The specific weight and the modulus of rigidity of the spring material are 7.35×10^{-5} N/mm^3 and 8×10^4 MPa, respectively. The ends of the spring are square and ground. Inspect the suitability of the spring for this engine. At what speed of the engine does the spring resonate? (14 Marks)

3. a. Show that in flat belt drives the ratio of belt tensions is given by = $\frac{T_1}{T_2} = e^{\mu\theta}$, where T_1 and T_2 are belt tensions, μ is the coefficient of friction and θ is the angle of wrap. (05 Marks)

b. Select the type and number of V-belts required to drive a crusher, which works 8 hours a day. The power transmitted is 65 kW. The motor shaft runs at 900 rpm and carries a pulley of 250mm diameter, the crusher shaft rotates at 300 rpm and the centre distance is 700 mm. Determine the pitch length of the belt required. (15 Marks)

4. The block brake shown in figure A4 (a) has two shoes each subtending an angle 80°; the coefficient of friction of the brake material is 0.3. The brake drum diameter is 800 mm while the rope drum measures 1200 mm in diameter. It is required to stop the load of 20 kN, lowering at a velocity of 5 m/s in a distance of 20 meters. Determine; the effort E required at the end of the lever, the width of the brake shoes given the permissible pressure of 0.7 MP*a* and the amount of heat generated. (20 Marks)

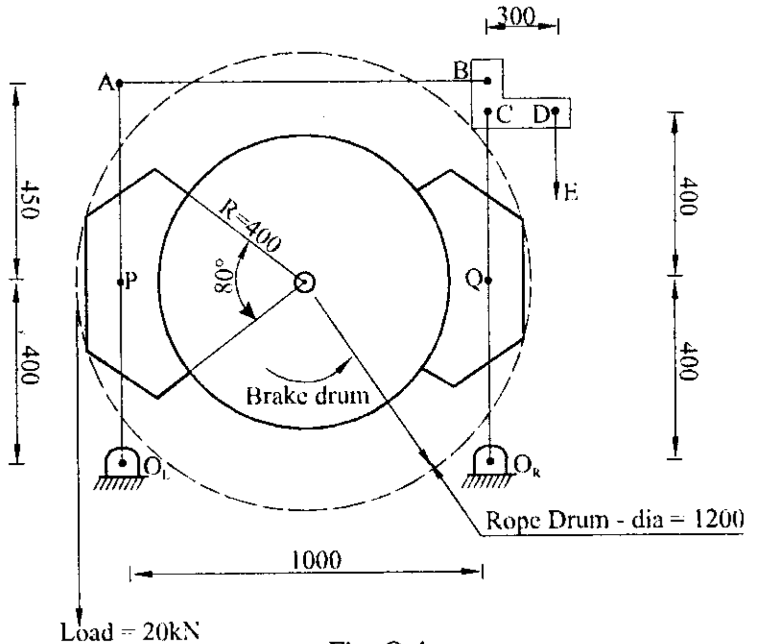

Fig. Q 4

5. a. Give reasons for the selection of involute tooth profile for gears more commonly. While generating less number of teeth on gear wheels the problem of interference prevails in involute tooth gearing, does such a problem really exist in cycloidal tooth gearing? (05 Marks)

b. A pair of spur gears is required to transmit 5 kW of power. The pinion rotates at a speed of 1620 rpm and the gear is required to run 420 rpm. Determine the least number of teeth on the gear and pinion such that velocity ratio does not deviate at all. The tooth from is 20° full depth involute, therefore the number of teeth selected on the pinion should not be less than the theoretical minimum. The permissible stress in the material of the gear and the pinion are 55 MPa and 65 MPa respectively. Design the gears for beam strength only and determine all the proportions of the gearing. (15 Marks)

6. a. Determine the cone pitch angles, pitch diameters for the following bevel gear pairs:
   i) For shaft angle 77° (Acute angle bevel gearing)
   ii) For shaft angle 147° (Obtuse angle bevel gearing)

   The module is 5 mm and the number of teeth on the pinion and the gear are 14 and 42, respectively. Draw the sketches of gearing. (05 Marks)

   b. A pair of herringbone gears is used to transmit 50 kW power. The pinion rotates at 2800 rpm. The number of teeth on the pinion and gear are 21 and 109, respectively. The tooth form is 20° full depth involute and the helix angle is 25°. The material for the gear is cast steel with a hardness of 150 BHN and for the pinion is steel. The wear and lubrication factor may be taken as 1.15. The normal module employed for the gears is 4 mm and the face width of the gears is 20 times the normal module. Determine the required hardness for the pinion for continuous operation of the drive. Also recommend the class of gears. (15 Marks)

7. a. Determine the proportions of a worm gear drive to transmit 15 kW power from a motor shaft rotating at 2880 rpm. The wheel shaft rotates at 240 rpm. The gear is made from phosphor bronze with permissible strength of 82.4 MPa and hardness 100 BHN, while the worm is made from hardened steel. Determine i) the efficiency of the drive and ii) whether the drive requires cooling. (20 Marks)

8. a. Derive the petroff's equation for frictional power loss of a lightly loaded journal rotating at high speed concentric to the bearing. (05 Marks)

   b. Determine the power loss in a bearing, the diameter of journal is 60 mm and length 80mm. The diametral clearance is 0.12 mm. The bearing supports a load of 5000 N and the journal rotates at a speed of 2500 rpm. The kinematic viscosity and the specific gravity of the oil used at the operating temperature of the bearing are 50 centi-stokes and 0.9, respectively.

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06ME61

Sixth Semester B.E. Degree Examination, June-July 2009

ME/IP/IM/MA/AU

Design of Machine Elements - II

Time : 3 Hours] [Max. Marks : 100

Note : *1. Answer any FIVE full questions.*
2. Use of Design Data Handbook is permitted.
3. Missing data may be suitably assumed.

PART - A

1. a. What are the assumptions made in finding stress distribution for a curved flexural member ? Also state two major differences between a straight beam and a curved beam. (05 Marks)

   b. Determine the value of web thickness 't' in the T – cross section of a curved beam shown in Fig.Q.1(b) such that the normal stresses due to bending at the extreme inner and outer fibres are numerically equal. (15 Marks)

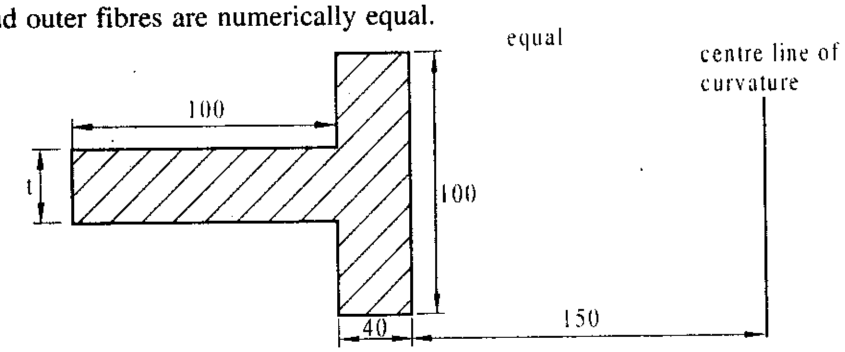

Fig. Q.1 (b)

2. a. A cast steel cylinder of 300 mm internal diameter is to contain liquid at a pressure of $12.5 N/mm^2$. It is closed at both ends by unstayed flat cover plates rigidly bolted to the shell flange. Determine the thickness of the cover plates if the allowable working stress for the cover material is $75 \ N/mm^2$. (05 Marks)

   b. Design a shrink fit joint to join two cylinders of diameter 150mm × 200mm and 200mm × 250mm. Maximum tangential stress in the components due to shrink fitting is to be limited to 40MPa. Also determine the axial force necessary to dis-engage the joint if the length of the joint is 200mm and the maximum power that can be transmitted at a rated speed of 1000 rpm. The cylinder material has a modulus of elasticity 210 GPa and Poisson's ratio 0.3. (15 Marks)

3. a. Derive the expression for the stress induced in helical coil spring. (05 Marks)

   b. Design a valve spring for an automobile engine, when the valve is closed, the spring produces a force of 45N and when it opens, produces a force of 55N. The spring must fit over the valve bush which has an outside diameter of 20mm and must go inside a space of 35mm. The lift of the valve is 6mm. The spring index is 12. The allowable stress may be taken as 330 MPa and modulus of rigidity, G 80 GPa. (15 Marks)

4. a. Define "Formative number of teeth" as applied to Helical gears and explain its importance in the design of Helical gears. (05 Marks)

   b. Design a pair spur gears to transmit 20 kW of power while operating for 8 to 10 hours per day sustaining medium shock, from a shaft rotating at 1000 rpm to a parallel shaft which is to rotate at 310 rpm. Assume the number of teeth on pinion to be 31 and 20° full depth involute tooth profile. The material for pinion is C40 steel, untreated whose $\sigma_0 = 206.81$ N/mm^2 and for gear is cast steel, 0.2% C, untreated whose

   $\sigma_0 = 137.34$ N/mm^2. Check the design for Dynamic load if Load factor, C = 522.464 N/mm and also for wear load taking Load stress factor, K = 0.279 N/mm^2. Suggest suitable hardness. (15 Marks)

PART - B

5. a. Under what circumstances the bevel gears are used ? Give a detailed classification of bevel gears. (05 Marks)

   b. Design a worm gear drive to transmit a power of 2kW at 1000 rpm. The speed ratio is 20 and centre distance is 200mm. Assume the number of teeth on worm wheel to be 40 and number of starts on worm to be 2. Assume hardened steel worm and phosphor bronze wheel for which $\sigma_0 = 55$ N/mm^2.

   Check the gear from stand point of strength and wear if load stress factor, K = 0.69 MPa. If the amount of Heat generated is 1.7 kW, check whether artificial cooling arrangement is necessary or not for a temperature rise of 40°K. (15 Marks)

6. a. A multiple disc clutch has five plates having four pairs of active friction surfaces. If the intensity of pressure is not to exceed 0.127 N/mm^2, find the power transmitted at 500rpm. The outer and inner radii of friction surfaces are 125 mm and 75 mm respectively. Assume uniform wear and take co-efficient of friction as 0.3. (05 Marks)

   b. A differential band brake as shown in Fig.Q.6(b), has an angle of contact of 225°. The band has a compressed woven lining and bears against a cast iron drum of 350mm diameter. The brake is to sustain a torque of 350 N.m. and the co-efficient of friction between the band and the drum is 0.3. Find:

   i) The necessary force, P for the clockwise and anticlockwise rotation of the drum and

   ii) The value of 'OA' for the brake to be self locking, when the drum rotates clockwise. (15 Marks)

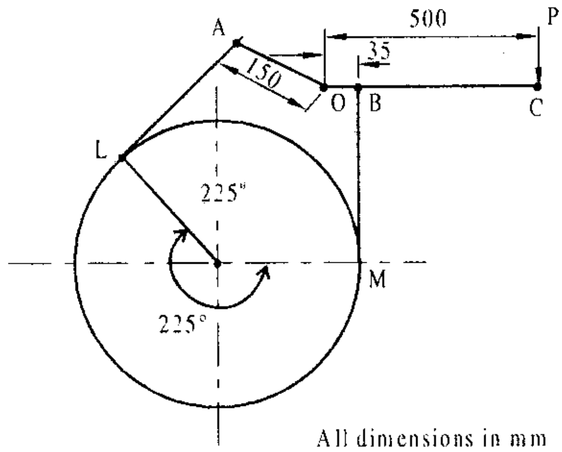

Fig. Q.6(b)

7. a. Derive Petroff's equation for a lightly loaded bearing. (05 Marks)

   b. Design a full journal bearing subjected to 6kN at 1000rpm of the journal. The journal is of hardened steel and the bearing is of babbit material. The bearing is operated with SAE40 oil at 70°C and the ambient temperature is 30°C. Also determine the amount of artificial cooling required. (15 Marks)

8. a. Derive the expression for the ratio of tensions in belt drive without considering the effect of centrifugal tension. (05 Marks)

   b. Two shafts 1 metre apart are connected by a V-belt to transmit 90kW at 1200 rpm of a driver pulley of 300mm effective diameter. The driver pulley rotates at 400 rpm. The angle of groove is 40° and the co-efficient of friction between the belt and the pulley rim is 0.25. The area of the belt section is 400mm^2 and the permissible stress is 2.1 MPa. Density of belt material is 1100kg/m^3. Calculate the number of belts required and the length of the belt. (15 Marks)

06ME61

Sixth Semester B.E. Degree Examination, Dec.09/Jan.10

ME/IP/IM/MA/AU

Design of Machine Elements - II

Time : 3 Hours] [Max. Marks : 100

Note : *1. Answer any FIVE full questions.*
2. Use of Design Data Handbook is permitted.
3. Missing data may be suitably assumed.

PART - A

1. a. Determine the maximum stress induced in a ring cross section of 50 mm diameter rod subjected to a compressive load of 20 kN. The mean diameter of the ring is 100 mm. (10 Marks)

   b. A cast iron cylinder of internal diameter 200 mm and thickness 50 mm is subjected to a pressure of 5 N/mm^2. Calculate the tangential and radial stresses at the inner, middle and outer surface. (10 Marks)

2. a. A railway wagon weighing 40 kN and moving with a speed of 10 km/hour has to be stopped by four buffer springs in which the maximum compression allowed is 200 mm. Find the number of turns in each spring of mean diameter 150 mm. The diameter of spring wire is 25 mm. Take $G = 82.7 \times 10^3$ MN/m^2. (10 Marks)

   b. A multi-leaf spring with camber is fitted to the chasis of a automobile over a span of 1.2 meter to absorb shocks due to a maximum load of 20 kN. The spring material can sustain a maximum stress of 0.4 GPa. All the leaves of the spring were to receive the same stress. The spring should have at least 2 full length leaves out of 8 leaves. The leaves are assembled with bolts over a span of 150 mm width at the middle. Design the spring for a maximum deflection of 50 mm. (10 Marks)

3. Design a pair of spur gears to transmit 20 kW from a shaft rotating at 1000 rpm to a parallel shaft which is to rotate at 310 rpm. Assume number of teeth on pinion 31 and 20° full depth tooth form. (20 Marks)

4. Design a pair of helical gears to transmit power of 15 kW at 3200 rpm with speed reduction 4 : 1 pinion is made of cast steel 0.4% C-untreated. Gear made of high grade CI. Helix angle is limited to 26° and not less than 20 teeth are to be used on either gear. Check the gears for dynamic and wear considerations. (20 Marks)

PART - B

5. a. A pair of bevel gears transmitting 7.5 kW at 300 rpm of pinion. The pressure angle is 20°. The pitch diameters of pinion and gear at their large ends are 150 mm and 200 mm respectively. The face width of the gears is 40 mm. Determine the components of the resultant gear tooth force and draw free body diagram of forces acting on the pinion and the gear. (10 Marks)

   b. A two teeth right hand worm transmits 2 kW at 1500 rpm to a 36 teeth wheel. The module of the wheel is 5 mm and the pitch diameter of the worm is 60 mm. The pressure angle is 14.5°. The co-efficient of friction is found to be 0.06.

   i) Find the centre distance, the lead and the lead angle.

   ii) Determine the forces. (10 Marks)

6. a. A single plate friction clutch of both sides effective has 0.3 m outer diameter and 0.16 m inside diameter. The co-efficient of friction is 0.2 and it runs at 1000 rpm. Find the power transmitted for uniform wear and uniform pressure distribution cases if the allowable maximum pressure is 0.08 MPa. (10 Marks)

   b. In a simple band brake, the length of lever is 440 mm. The tight end of the band is attached to the fulcrum of the lever and the slack end to a pin 50 mm from the fulcrum. The diameter of the brake drum is 1 m and arc of contact is 300°. The co-efficient of friction between the band and the drum is 0.35. The brake drum is attached to a hoisting drum of diameter 0.65 m that sustains a load of 20 kN. Determine

   i) Force required at the end of lever to just support the load.

   ii) Width of steel band if the tensile stress is limited to 50 N/mm^2. (10 Marks)

7. a. Derive Petroff's equation, with usual notations. (10 Marks)

   b. A lightly loaded bearing of 70 mm long and 70 mm in diameter is acted on by 1.5 kN radial load. The radial clearance is 0.07 mm and the journal is rotating at 25000 rpm. The viscosity of the oil is 3.45×10^{-3} pa.s. Determine frictional power loss using Petroff's equation. (10 Marks)

8. a. A compressor is driven by 900 rpm motor by means of 250 mm × 10 flat belt. The motor pulley is 0.3 m in diameter and the compressor pulley is 1.5 m diameter. The distance between the centres of the pulleys is 2 m. A jockey pulley is used to make the angle of wrap on the smaller pulley 220° and the larger pulley 270°. The coefficient of friction between the belt and the smaller pulley is 0.3 and between the belt and the larger pulley is 0.22. The maximum allowable belt stress is 2 MN/m^2 and the specific weight of the belt material is 9.515 kN/m^3. Determine the power that can be transmitted by the belt drive. (10 Marks)

   b. A compressor requiring 90 kW is to run at 250 rpm. The drive is by V-belt from an electric motor running at 750 rpm. The diameter of the pulley on the compressor shaft is 1 m, while the center distance between the pulleys is limited to 1.75 m. The belt speed should not exceed 1600 m/min. Determine the number of V-belts required to transmit the power if each belt has a cross sectional area of 375 mm^2 and density of 1 Mg/m^3 and the coefficient of friction between the belt and the pulley is 0.25.(10 Marks)